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INTERNATIONAL WORKSHOP ON ADVANCES IN ANALYTICAL METHODS IN AERODYNAMICS

Program and Abstracts

July 12–14, 1993

Hotel Amber Baltic
Miedzyzdroje, Poland

INTERNATIONAL WORKSHOP ON ADVANCES IN ANALYTICAL METHODS IN AERODYNAMICS

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Hotel Amber Baltic
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INTERNATIONAL WORKSHOP ON

ADVANCED IN MODELING OF AERODYNAMIC FLOWS

Hotel Amber Baltic
Miedzyzdroje, Poland
July 12 - 14, 1993

Co-Chairs

Julian D. Cole	Rensselaer Polytechnic Institute Troy, New York, USA
Vladimir V. Sychev	Central Aerohydrodynamic Institute (TsAGI) Zhukovsky, Moscow Region, Russia

Organizing Committee

Mark Barnett	United Technologies Research Center, East Hartford, Connecticut USA
Alfred Kluwick	Technische Universität Wien, Austria
Alric P. Rothmayer	Iowa State University, Iowa, USA
Anatoly I. Ruban	Central Aerohydrodynamic Institute (TsAGI) Zhukovsky, Moscow Region, Russia
F. T. Smith	University College London, London, United Kingdom
J. D. A. Walker	Lehigh University, Bethlehem, Pennsylvania, USA

Acknowledgements

The Organizing Committee would like to thank Dr. Roy E. Reichenbach for support of this meeting through the European Research Office of the United States Army. The financial support of the European Office of Aerospace Research and Development through Dr. Wladimiro Calarese is also gratefully acknowledged. Critical financial support for the meeting was provided by the William and Mary Greve Foundation. Thanks are also due to the National Science Foundation who provided travel support for the U. S. participants.

Travel grants were provided to Russian participants by United Technologies Corporation whose assistance was a key factor in making this meeting a success. The Organizing Committee would like to express deep appreciation to Dr. M. J. Werle of United Technologies Research Center who conceived of this Workshop originally and who provided continuous encouragement and support.

The Organizing Committee would also like to thank Mrs. JoAnn Casciano for her patient and careful typing of the abstracts.

CONFERENCE SCHEDULE

MONDAY, July 12

8:30 Opening and Announcements

8:35 Welcoming remarks

Julian D. Cole (Rensselaer Polytechnic Institute, Troy, NY, USA)

V.V. Sychev (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

8:45 Opportunities for East/West Collaboration (J.D.A. Walker)

SESSION 1. Chairman: J.D.A. Walker (Lehigh University, Bethlehem, PA, USA)

9:00 J.S.B. Gajjar (U. of Manchester, United Kingdom)

The nonlinear development of cross-flow vortices in compressible boundary layers

9:15 Alexander B. Lessen and Vladimir U. Lunev (Central Research Institute of Machine Building, Kalingrad, Russia)

Abnormal peaks of increased heat transfer on the blunted delta wing in hypersonic flow

9:30 J.A. Steketee (Delft University, The Netherlands)

Theoretical aspects of the Lagrangian equations of motion for the unsteady rectilinear flow of an ideal gas

SESSION 2. Chairman: E.D. Terent'ev (Computing Center, Russian Academy of Sciences, Moscow, Russia)

9:45 Yu. B. Lifshitz (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

The wind tunnel transonic stabilization law

10:00 M. Barnett (United Technologies Research Center, East Hartford, CT, USA)

Viscid/inviscid interaction analysis of turbomachinery cascade flows

10:15 S.V. Manuilovich (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

On new methods of delaying laminar-turbulent transition in sound-exposed boundary layers

10:30 *Coffee Break*

SESSION 3. Chairman: Igor I. Lipatov (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

11:00 S.O. Seddougui (University of Birmingham, United Kingdom)

Fastest growing Görtler vortices in compressible boundary layers

11:15 V.V. Bogolepov (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Asymptotic theory of Görtler vortices

11:30 Lennart S. Hultgren (NASA Lewis Research Center, Cleveland, Ohio, USA)

Oblique waves interacting with a nonlinear plane wave

SESSION 4. Chairman: A.P. Rothmayer (Iowa State University, Ames, Iowa, USA)

11:45 Valery N. Golubkin (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Theory of three-dimensional hypersonic flow over a wing at moderate angles of attack

12:00 J.W. Elliott (University of Hull, United Kingdom)

The influence of surface cooling on compressible boundary-layer stability

12:15 E.D. Terent'ev (Computing Center, Russian Academy of Sciences, Moscow, Russia)

A linear problem of a vibration in a boundary layer on a partially elastic surface elastic surface

12:30 *Lunch*

SESSION 5. Chairman: V.V. Sychev (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

2:30 A.G. Walton, R.I. Bowles and F.T. Smith (Imperial College and University College London, United Kingdom)

Vortex wave interaction in a strong pressure gradient

2:45 M.A. Brutyan and P.L. Krapivsky (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Weak turbulence in periodic compressible flows

3:00 H.K. Cheng (University of Southern California, CA, USA)

On extension of continuum models to rarefied gas dynamics

3:15 V.I. Zhuk (Computing Center, Russian Academy of Sciences, Moscow, Russia)

Soliton disturbances in a transonic boundary layer

SESSION 6. Chairman: F.T. Smith (University College London, United Kingdom)

3:30 H. Herwig (Ruhr-Universität Bochum, Germany)

The influence of temperature and pressure on boundary layer stability

3:45 A.L. Gonor (Moscow University, Moscow, Russia)

Nonlinear asymptotic solutions of flow past thin bodies on entry into a compressible fluid

4:00 A.E.P. Veldman (Groningen, The Netherlands)

Curvature effects and strong viscous-inviscid interactions

4:15 L.V. Ovsiannikov (Institute of Hydrodynamics, Siberian Division of the Russian Academy of Sciences, Novosibirsk, Russia)

4:30 *Coffee Break*

POSTER SESSION 1 Chairman: M. Barnett (United Technologies Research Center, East Hartford, CT, USA)

5:00 M.A. Kravtsova (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Numerical solution for a criss-cross interaction problem

V.N. Diesperov (Computing Center, Russian Academy of Sciences, Moscow, Russia)

The behaviour of self-similar solutions of boundary layers with zero pressure gradient

I.A. Chernov (Saratov State University, Saratov, Russia)

Parametric representation of exact solutions of the transonic equations

G.N. Dudin (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Three-dimensional laminar boundary layer on a finite delta wing in viscous interaction with hypersonic flow

Gregory Vylensky (Krylov Shipbuilding Research Institute, St. Petersburg, Russia)

Three-dimensional breakaway model of a free streamline type

C.N. Zhikharev (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Separation phenomena in a hypersonic flow with strong wall cooling: subcritical regime

7:30-9:00 *Dinner (Buffet)*

TUESDAY, July 13

8:25 Announcements

SESSION 7. Chairman: V. Ya. Neiland (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

8:30 A.F. Messiter (University of Michigan, USA)

Nonlinear instability of supersonic vortex sheets and shear layers

8:45 A.M. Gaifullin and S.B. Zakharov (Central Aerohydrodynamic Institute (TsAGI) Zhukovsky, Russia)

Calculation of separated flow on a circular cone with viscous-inviscid interaction

9:00 N.D. Malmuth (Rockwell International Science Center, Thousand Oaks, CA, USA)

Unsteady hypersonic thin shock layers and flow stability

9:15 N.G. Kuznetsov (Institute for Engineering Studies, Russian Academy of Sciences, St. Petersburg, Russia)

Wave resistance of a submerged body moving with an oscillating velocity

SESSION 8. Chairman: A.E.P. Veldman (University of Groningen, The Netherlands)

9:30 S.P. Fiddes (University of Bristol, United Kingdom)

Vortical flow past slender bodies at incidence: existence, uniqueness, bifurcation, and stability

9:45 A.G. Kuz'min (St. Petersburg University, Russia)

Weakening of the cumulative phenomenon and shocks in transonic flows

10:00 A.T. Degani (Lehigh University, Bethlehem, PA, USA)

Structure of the three-dimensional turbulent boundary layer

10:15 E.V. Bogdanova-Ryzhova and O.S. Ryzhov (Computing Center, The Russian Academy of Sciences, Moscow, Russia)

On the Landau-Goldstein singularity and marginal separation

10:30 *Coffee Break*

SESSION 9. Chairman: Yu. B. Lifshitz (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

11:00 Julian D. Cole, M.C. Kropinski and D. Schwendeman (Rensselaer Polytechnic Institute, Troy, NY, USA)

Construction of optimal critical airfoils

11:15 A. Kluwick, Ph. Gittler and R.J. Bodonyi (Technische Universität Wien, Austria and Ohio State University, Columbus, OH, USA)

Viscous-inviscid laminar interaction near the trailing tip of an axisymmetric body

11:30 A.I. Ruban (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

The generation of Tollmein-Schlichting waves by free-stream turbulence

SESSION 10. Chairman: A. Kluwick (Technische Universität Wein, Austria)

11:45 S.J. Cowley and X. Wu (University of Cambridge and Imperial College, United Kingdom)

Nonlinear modulation of instability modes in shear flows

12:00 A.V. Fedorov and A.P. Khokhlov (Moscow Institute of Physics and Technology, Moscow, Russia)

Receptivity of a supersonic boundary layer to sound near the leading edge of a flat plate

12:15 P.Y. Lagrée (Université Paris, France)

Influence of the entropy layer on viscous triple deck hypersonic scales

SESSION 11. Chairman: A.I. Ruban (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

2:30 V. Zoubtsov and G.G. Soudakov (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Asymptotic solution of an axisymmetric ideal-flow problem for a cavity apex region

2:45 R. Puhak, A.T. Degani and J.D.A. Walker (Lehigh University, USA)

Separation and heat transfer upstream of obstacles

3:00 Victor V. Sychev (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Asymptotic theory of vortex breakdown

3:15 S.N. Timoshin and F.T. Smith (University College London, United Kingdom)

On the nonlinear vortex-Rayleigh wave interaction in a boundary-layer flow

SESSION 12. Chairman: J.D. Cole (Rensselaer Polytechnic Institute, USA)

3:30 L.L. Van Dommelen (Florida State University, USA)

Lagrangian computation of 3D unsteady separation

3:45 A.F. Sidorov (Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Ekaterinburg, Russia)

Analytic methods for the study of adiabatic compression of a gas

4:00 Rowena G.A. Bowles, Bharat T. Dodia and F.T. Smith (University College London, United Kingdom)

Aspects of transitional-turbulent spots in boundary layers

4:15 A.P. Rothmayer, R. Bhaskaran and D.W. Black (Iowa State University, Ames, Iowa, USA)

On two-dimensional, incompressible laminar boundary layer separation near airfoil leading edges

4:30 *Coffee Break*

POSTER SESSION 2: Chairman: Oleg S. Ryzhov (Computing Center, Russian Academy of Sciences, Moscow, Russia)

5:00 A.P. Khokhlov (Moscow Institute of Physics and Technology, Zhukovsky, Russia)

Asymptotic model of triad evolution in boundary layers

V.S. Sadovsky (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

On drag and thrust forces in an ideal fluid flow with constant vorticity

N.S. Bakhvalov and M.E. Eglit (Moscow State University, Moscow, Russia)

Asymptotic analysis of small-disturbance propagation in mixtures

V.N. Trigub and S.E. Grubin (INTECO srl, Frosinone, Italy)

The asymptotic theory of hypersonic boundary-layer stability

V.B. Zametaev (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Thin shock layer theory with interaction: marginal regime

I.G. Fomina (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Unsteady flow on the leading edge of an oscillating airfoil

7:30-9:00 *Dinner-Buffet*

WEDNESDAY, July 14

8:25 Announcements

SESSION 13. Chairman: S.C.R. Dennis (University of Western Ontario, Canada)

8:30 A.T. Conlisk (Ohio State University, USA)

An asymptotic approach to vortex-body collisions

8:45 Igor I. Lipatov (Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia)

Study of nonstationary processes of a strong viscous-inviscid interaction

9:00 Thomas C. Adamson (University of Michigan, USA)

Viscous effects on critical flow in the exit region of a thin channel

9:15 Igor V. Savenkov (Computing Center, Russian Academy of Sciences, Moscow, Russia)

Resonant interactions and solitons in inlet pipe flow

SESSION 14. Chairman: A.G. Kulikovsky (Mathematical Steklov Institute, Moscow, Russia)

9:30 L. Pamela Cook and G. Schleiniger (University of Delaware, USA)

Some axisymmetric transonic flows

9:45 Yu. Shmyglevsky (Computing Center, Russian Academy of Sciences, Moscow, Russia)

On solutions of both the Euler and the Navier-Stokes equations

10:00 S.C.R. Dennis (University of Western Ontario, Canada)

Boundary-layer models for flow past a cylinder

10:15 S.I. Chernyshenko (Moscow University, Russia)

High Reynolds number structure of steady two-dimensional flow through a row of bluff bodies

10:30 *Coffee Break*

SESSION 15. Chairman: T.C. Adamson (University of Michigan, USA)

11:00 R.G.A. Bowles and F.T. Smith (University College London, United Kingdom)

Weakly and fully nonlinear effects in channel flow transition: an experimental comparison

11:15 A.G. Kulikovsky (Mathematical Steklov Institute, Moscow, Russia)

On transition to instability in flows that depend on a slowly-varying spatial variable: stability of pipe flow

11:30 J.Ph. Brazier, B. Aupoix and J. Cousteix (ONERA/CERT, Toulousa, France)

Asymptotic equations for the boundary layer using a defect formulation

SESSION 16. Chairman: L.V. Ovsiannikov (Siberian Division of the Russian Academy of Sciences, Novosibirsk, Russia)

11:45 V.N. Trigub, A.B. Blokhin and I.N. Simakin (INTECO srl, Frosinone, Italy)

Asymptotic study of dissipation and breakdown of a wing-tip vortex

12:00 F. Stephan and E. Deriat (ONERA, Chatillon, France)

Energy transfer from a turbulent boundary layer mean flow to 3D large scale waves

12:15 G.L. Korolev (Central Aerohydrodynamic Institute (TsAGI), Russia)

Flow separation and non-uniqueness of boundary-layer solutions

12:30 *Lunch*

SESSION 17. Chairman: A.F. Sidorov (Ural Branch of the Russian Academy of Sciences, Ekaterinburg, Russia)

2:30 K. Cassel and J.D.A. Walker (Lehigh University, USA)

Viscous-inviscid interactions in unsteady boundary-layer separation

2:45 M.V. Ustinov (Central Aerohydrodynamic Institute (TsAGI), Russia)

Generation of secondary instability modes by Tollmein-Schlichting wave scattering from uneven walls

3:00 M. E. Goldstein (NASA Lewis Research Center, Cleveland, OH, USA)

Oblique instability waves in nearly parallel shear flows

SESSION 18. Chairman: M.J. Werle (United Technologies Research Center, East Hartford, CT, USA)

3:15 Panel Discussion

J. D. Cole, S.P. Fiddes, A.I. Ruban, F.T. Smith, J.D.A. Walker

4:30 *Coffee Break*

5:00 Cooperative Research Seminar (J.D.A. Walker)

8:00 *Conference Banquet*

THE NONLINEAR DEVELOPMENT OF CROSS-FLOW VORTICES IN COMPRESSIBLE BOUNDARY LAYERS

by

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This paper is concerned with the nonlinear spatial/temporal development of both stationary as well as non-stationary cross-flow vortices in compressible boundary layers. Unsteady nonlinear critical layer theory is used to study first the linear stability properties of disturbances in a fully three-dimensional boundary flow, and this is then followed by an investigation of the nonlinear effects.

After suitable approximations the problem for long-wavelength non-stationary vortices can be shown to be very similar to that for the evolution of oblique modes in planar boundary layers, Gajjar (1993). The mean boundary layer profile in the cross-flow direction gives rise to a problem involving the solution of two coupled unsteady nonlinear critical layer equations at the upper and lower critical layers. The amplitude of the cross-flow vortex is directly coupled to the unsteady jumps arising across both critical layers and this interplay, and interaction with the critical layer dynamics strongly influences the nonlinear development of the vortex. The numerical solution of this coupled problem is currently in progress and will be described.

The theory for the nonlinear development of stationary cross-flow vortices is different in that the nonlinear critical layer dynamics is coupled with the properties of an unsteady wall layer. This problem is quite involved although some special cases can be studied, and these will be discussed.

Finally, it will be shown how the analytical results from the linear (and nonlinear) stability analysis can be used to compute growth rate curves for three-dimensional compressible boundary layer flows. The analytical results compare very favourably with full numerical solutions of the linear stability equations and provide a more versatile tool for exploring the parametric dependence of Mach number, heat transfer etc. on the stability properties of the base flow.

ABNORMAL PEAKS OF INCREASED HEAT-TRANSFER ON THE BLUNTED DELTA WING IN HYPERSONIC FLOW

by

Alexander B. Lessen
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and

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In hypersonic flow, the strong influence of a small nose bluntness on the laminar heat-transfer pattern on the windward surface of thin long wings was discovered in the experiments of Gubanova et al. (1992). This effect consists of two narrow longitudinal bands showing increasing heat-transfer rate. These strips are almost parallel to each other and close to the center line of the wing; they start at the point (or region) A of the impingement of the strong front bow shock (caused by the nose bluntness) at the leading edge of the wing, as shown in Figure 1. The same result was also obtained by solving the Navier-Stokes equations. This phenomenon is a purely hypersonic one. With decreasing Mach number M from 14 to 8, it weakens substantially and becomes appreciable only with an increase of the wing half-angle (from the usual 15 up to 25 degrees). Moreover, it was not observed at $M = 6$ at all. The maximum effect is realized at an angle of attack about 10, and it has a tendency to vanish at decreasing of angle of attack to 0 or at angles beyond 15 degrees.

Attempts at explanation of this effect are presented below. Previously known cases of the appearance of the local heat-transfer rate peaks are caused, as a rule, by a local maximum of pressure, but here the strips occur at nearly constant pressure, equal approximately to that on a sharp wing. Therefore these strips must arise from another cause.

We connect them with a specific region of flow divergence which we will refer to as the inertial one due to interaction of the front shock with the leading edge. In these cases, the "impingement spot" of local pressure takes place in the vicinity of the point A. This interaction region is observed for a sharp leading edge, provided that the front bow shock is axisymmetric as for a blunted cylinder. There is a discontinuity in the flow parameters in this region, and its break-up forms the spot mentioned above. The gas passing this spot expands to the wing pressure level and then spreads out in a longitudinal direction far from the origination. This leads to the formation of a bundle of divergent stream lines, a decrease of the boundary-layer thickness and, consequently, a rise in the heat-transfer rate. Owing to the constant pressure, the stream lines in this bundle are nearly straight, and the three-dimensional heat transfer law gives almost constant heat-transfer increase ratio, close to 1.7 for a cone to wedge, as is seen from Figure 1 for the data at a large distance from the nose.

References

Gubanova, O. I., Zemliansky, B. A., Lessin, A. B., Lunev, V. V., Nikulin, A. N. and Susin, A. V. 1992 "Anomalous Heat-Transfer on the Windward Surface of the Triangular Wing with Blunted Nose in the Hypersonic Flow", Collection of Reports Aerospace Aircraft Aerodynamics of the Annual Workshop-School of the Central Aerohydrodynamics Institute, Part I, Zhukovsky, Moscow Region, Russia.

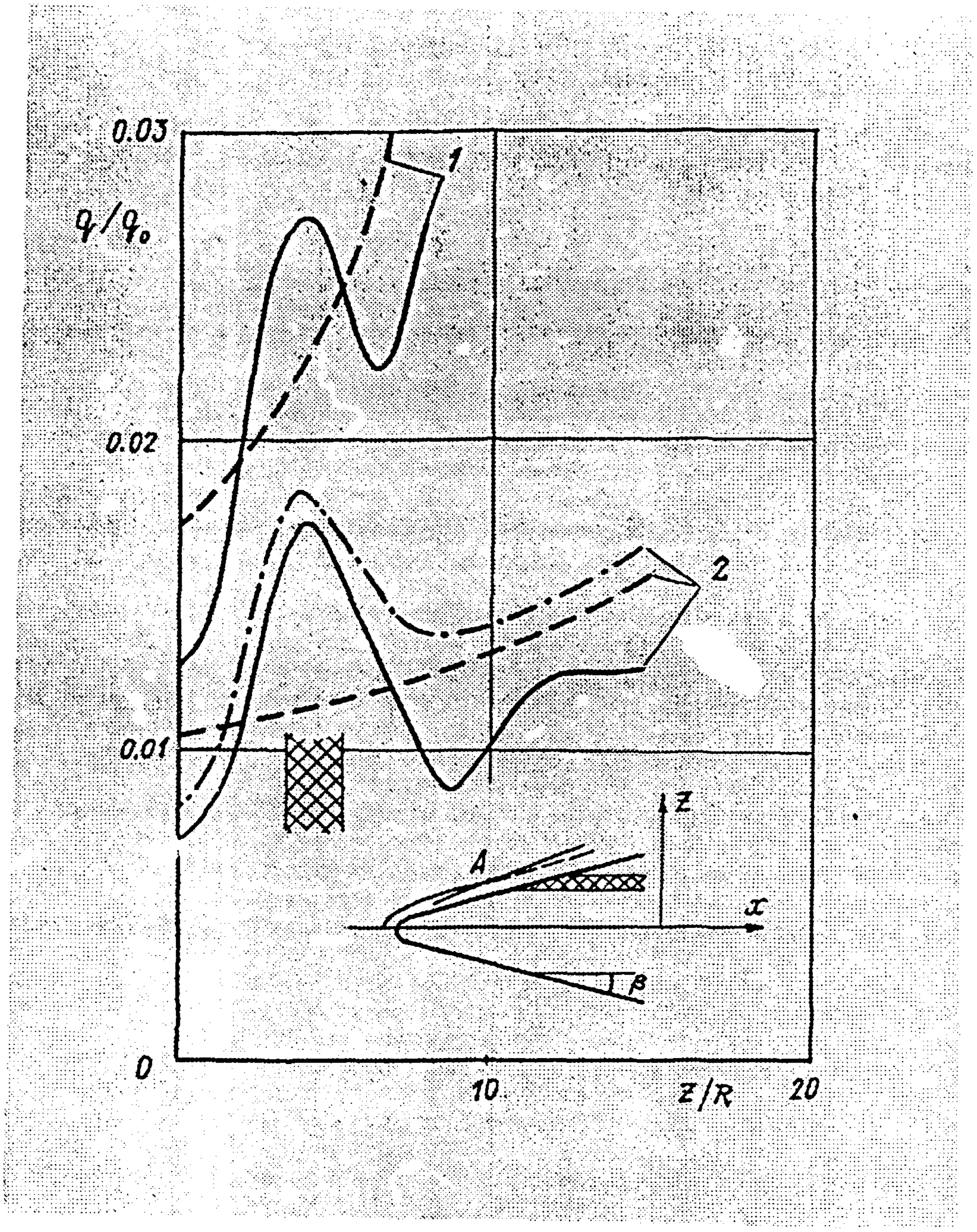

Figure 1. Heat-transfer on the wing with a nose bluntness of radius R
1 – x/R = 37, 2 – x/R = 100
——- for blunt leading edges of radius 0.5R
— - for sharp leading edges
------ for sharp nose and leading edges

THEORETICAL ASPECTS OF THE LAGRANGIAN EQUATIONS OF MOTION FOR THE UNSTEADY RECTILINEAR FLOW OF AN IDEAL GAS

by

J. A. Steketee
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Delft University of Technology
The Netherlands

1. The paper is occupied with the unsteady, one-dimensional (rectilinear) flow of an ideal gas with constant specific heats c_p and c_v, while neglecting effects of viscosity and heat conduction. The equations of motion for these problems are usually taken in the Eulerian form with the independent variables (x,t), but here we take the Lagrangian form with the independent variables (h,t), h the Lagrangian mass coordinate and t the time. The equations of motion then take the form

$$\frac{\partial V}{\partial t} - \frac{\partial u}{\partial h} = 0, \quad \frac{\partial u}{\partial t} + \frac{\partial p}{\partial h} = 0, \quad pV^\gamma = B(h) = b(h)^\gamma, \tag{1.1}$$

with u the velocity, p the pressure, V the specific volume, while $B(h) = b(h)^\gamma$ determines the entropy distribution in the gas. The gas is called homentropic when B(h) is a constant, non-homentropic when B(h) is h-dependent.

In the first part of the paper a systematic study of the equations (1.1) is made by introducing the analogous of 'potentials' and 'stream functions' together with their Legendre Transformations. In this way and with little effort, a survey is obtained of the simplest forms of the equations of motion. Several of these equations have appeared in the literature for special problems, usually after a good deal of not very transparent manipulation. Also second order potentials are introduced. A very simple form appears for the second order potential $\Phi(h,t)$. The problem then reduces to solving the equation

$$\phi_{tt}(\phi_{hh})^\gamma + B(h) = 0, \tag{1.2}$$

with

$$V = \phi_{hh}, \ u = \phi_{ht}, \ p = -\phi_{tt} \tag{1.3}$$

and subscripts denoting partial derivatives.

2. The second part of the paper begins with constructing the characteristic equations of the set (1.1).

For a homentropic gas, the classical Riemann invariants

$$r = u + \frac{2}{\gamma - 1}\alpha, \quad s = u - \frac{2}{\gamma - 1}\alpha$$

are obtained. It is then shown that for a non-homentropic gas with

$$B(h) = B_o{}^{-(3\gamma-1)}, \tag{2.1}$$

(generalized) Riemann-invariants r^* and s^* exist with

$$r^* = K + h(u + \frac{2a}{\gamma - 1}), \quad s^* = K + h(u - \frac{2}{\gamma - 1}\,\alpha), \tag{2.2}$$

where K is a potential for the momentum equation in (1.1).

The gas with B(h) given by (2.1) is called the LMS-gas after G. S. S. Ludford, M. H. Martin and K. P. Stanyukovich.

Two simple examples of the flow of an LMS-gas are discussed. These are

(i) a flow with $r^* = \text{constant}$, $s^* = \text{constant}$,

(ii) two domains of type (i) flow, separated by a normal shock-wave.

It is finally shown that the flow of LMS-gas may be obtained from the flow of a homentropic gas by applying a simple transformation, due to K. P. Stanyukovich (1954). In the first instance, the transformation applies to the mass coordinate h and the second order potential Φ, but the transformation rules for the other parameters u, p, V etc. are also deduced. Some other restrictions, for example that h should be positive, are discussed and may be removed.

3. The Transformation of Stanyukovich generates the flow of an LMS-gas, once a flow of a homentropic gas has been given. In the third part of the paper, the Transformation of Stanyukovich is generalized to the form of a 3-parameter continuous transformation group, which is closely related to the projective group on a line. The transformation rules for the different parameters of the problem are constructed. In particular it is found that the transformation rules for the Riemann invariants r, s, r^* and s^* are linear transformations. This may be of some interest for the representation of the group in contrast with the realization of the group. Some simple applications of the Transformation group will be presented.

Finally the three infinitesimal operators of the G_3 are constructed together with several extended operators. They are transformations close to the identity transformation and generate small perturbations to a given flow, which are contained within the group manifold. Partial differential equations to be satisfied by these perturbations can be written down, and the proper infinitesimal increments, when substituted in these equations, reduce them to identities. It will be illustrated by some simple examples.

THE WIND TUNNEL TRANSONIC STABILIZATION LAW

by

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It is well known that the local Mach number distribution on an airfoil surface in transonic flow is a weak function of the Mach number at infinity. This so-called "stabilization" law was established experimentally at various times in many countries. One can find a formulation with numerous examples in Holder's lecture delivered in commemoration of Reynolds and Prandtl in 1964, or in a paper by Krishtianovich et al. (1948), which was subsequently reprinted in a collection of the scientific papers of Krishtianovich collection. Later this law was generalized by Diesperov, Lifshitz and Ryzhov to unbounded flows using a transonic method of singular disturbances.

One of the wind tunnel stabilization law explanations was cast by Khristianovich et al. (1948) in terms of a blocking effect. If the law was valid, the airfoil surface Mach number distribution would be independent of upstream conditions. But in the unbounded flows the dependence exists and is governed by the flow characteristics at a large distance from the body. Because the far fields of the bounded and unbounded flows differ appreciably, quantitative expressions of the stabilization law can be different in these cases. The asymptotic estimation of how flow parameters on an airfoil in a wind tunnel test section with porous walls depend on the von Karman transonic similarity parameter is the topic of this paper.

The problem is solved by the singular disturbance method which has been developed in the study of unbounded flow at the Mach numbers near unity and in related problems. It is assumed that the test section can be simulated by an infinite two dimensional channel with parallel porous walls. Let (x, y) denote Cartesian coordinates with x measured along the channel and $y=0$ being the channel center line. An airfoil placed along $y=0$ has length L and the relative thickness $\tau \ll 1$. The distance between the walls is 2H, and the velocity of oncoming gas differs slightly from the critical velocity a_*. In accordance with the transonic small-perturbation theory (which can be used under these assumptions), a perturbation potential relative to a uniform sonic flow along the x axis is represented in the form

$$\Phi(x, y) = a_* L^{2/3}\tau\Phi'(x', y'; t) + \dots, \qquad x' = x/L, \qquad y' = y\tau^{1/3}/L. \tag{1}$$

The principal term in the expansion of $\Phi'(x', y'; t)$ with respect to τ satisfies the von Karman equation

$$-(\kappa+1)\Phi_x\Phi_{xx} + \Phi_y = 0, \tag{2}$$

in which κ is Poisson's adiabatic exponent and the primes on all variables here and below are omitted. On the airfoil $Y = \tau L y(x)$, the solution of equation (2) satisfies the no-flow condition transferred to the interval $0 < x < 1$ of the axis. The Darcy linear condition is usually used on the permeable walls at $Y = +\tau^{2/3}H/L$. Its compressible counterpart is of the form

$$\Phi_y \pm \mu(K + \Phi_x)K^{1/2} = 0, \qquad K = 2(\kappa+1)^{-1}\tau^{-2/3}(1 - M_\infty) \tag{3}$$

where μ is a constant. This relation is valid for walls having a transverse slit when $K \gg 1$, but apparently becomes meaningless in the limit $K \to 0$. An attempt to obtain the limiting equation at $K = 0$, which is a local relation satisfying the similarity transformation (1) and is exact for supersonic velocity, leads to

$$\Phi_y \pm \mu \, |\Phi_x|^{3/2} \, \mathrm{sign}(\Phi_x) = 0. \tag{3}$$

In the case $K \ll 1$, the potential is represented as the sum

$$\Phi = \varphi(x, y; \mu, \tau^{1/3} \, H/L) + \delta(K) \, \psi(x, y; \mu, \tau^{1/3} \, H/L),$$

in which φ is the potential of the flow in channel for $K = 0$, δ is a small parameter that tends to zero as $K \to 0$, and ψ is a function arising from the variation of the boundary condition on the wall when K departs from zero. At $K = 0$ there is always a sonic point O on the tunnel wall, at which the sonic line begins with the arrival of the airfoil. According to the singular disturbance method, we take O as a characteristic point of the problem and express φ as a series in distance from 0. If equation (3) is valid and $\mu = 0$, the principal term of the potential φ expansion in the neighborhood of this point is described by a self-similar solution of equation (2)

$$\varphi = y^{3n-2} \, f_o(\zeta), \quad \zeta = (\kappa + 1)^{-1/2} \, xy^{-n}, \quad n = 3, \tag{4}$$

in a local Cartesian coordinate system with origin at the point O. The function $f_o(\zeta)$ satisfies a second-order nonlinear differential equation. A first integral may be found and the behavior of the solution is completely investigated. The results of the analysis show that a solution in the form (4) exists, is unique, and analytic along the characteristic C_-^* that arrives at the point O from the airfoil. Along the characteristic C_+^*, there are discontinuities of the fourth derivatives. Shock waves are not present in the solution.

Outside a certain neighborhood of the point O, the function $\psi(x, y)$ can be represented in the form

$$\psi = y^{7-\nu} g_\nu(\zeta) + \ldots, \qquad \nu > 0, \tag{5}$$

in which $g_\nu(z)$ satisfies the homogeneous second-order linear equation

$$(f_o' - 9\zeta^2) \, g_\nu'' + \left[f_o'' + 6(5-\nu)\zeta\right] g_\nu' - (7-n)(6-n) \, g_\nu = 0. \tag{6}$$

The integrals of equation (6) satisfying the condition (3) pass through the singular point $\zeta = \zeta_1$ where $f_o' = 9\zeta^2$. Here, one linearly independent solution of equation (6) is an analytic function, and the other increases proportional to $(\zeta - \zeta_1)^{-2}$; the coefficients of these solutions are functions of ν. Therefore, ν must be chosen such that the coefficient of the solution which increases without bound at $\zeta = \zeta_1$ is zero. As a result, we obtain the required spectrum $\nu = 12, 24, \ldots$.

To establish the dependence $\delta(K)$, we make a transition from the present variables to dimensional ones by the transformation (1). Because equations (2) and (3) are invariant under this operation, the same product $\tau^k L^m$ can appear in the transformed expressions for φ and $\delta(K)\psi$. When using this for their principal terms given by equations (4) and (5), we obtain the required relation $\delta(K) = K^3$, which is the same as in the unbounded flow over an airfoil. This equation is a mathematical formulation of the stabilization law in the test section of a wind tunnel with porous walls.

VISCID/INVISCID INTERACTION ANALYSIS OF TURBOMACHINERY CASCADE FLOWS

by

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Efficient, accurate, steady analyses for predicting strong inviscid/viscid interaction(IVI) phenomena such as viscous-layer separation, shock/boundary-layer interaction and trailing-edge/near-wake interaction in blade passages are needed to predict the performance of turbomachinery cascades, particularly at off-design operating conditions. Two such analyses have been developed and are described in this talk. Both use an inviscid/viscid interaction approach, wherein high Reynolds number flow is assumed, allowing the flowfield to be divided into two regions: an outer inviscid region, and an inner or viscous-layer region, which is governed by Prandtl's equations. The two analyses differ principally in the approach used to predict the outer inviscid flow field. One technique assumes that the flow is governed by the Euler equations, which permits strong shocks and rotational flow, and the other method assumes potential flow. Finite-difference methods are used for the viscous-layer analyses and for the potential inviscid analysis, and a finite-volume approach is used to solve the Euler equations. The inviscid and viscid solutions are coupled using a semi-inverse global iteration procedure which permits the prediction of boundary-layer separation and other strong-interaction phenomena. Results for several cascades covering a wide range of inlet flow conditions are discussed, including conditions leading to large-scale flow separation, and comparisons with Navier-Stokes solutions and experimental data are shown. The factors that currently limit the accuracy of the IVI approach are also briefly discussed.

ON NEW METHODS OF DELAYING LAMINAR-TURBULENT TRANSITION IN SOUND-EXPOSED BOUNDARY LAYERS

by

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It is well known that sound can be an unfavorable influence in reducing the laminar portion of aerodynamic boundary layers. Reflecting from the wing surface, sound generates unstable waves (Tollmien-Schlichting waves), which undergo linear amplification and nonlinear interactions, and eventually lead to turbulence. Acoustic disturbances excite Tollmien-Schlichting waves only via scattering by longitudinal inhomogeneities in the boundary-layer flow, such as in the vicinity of the wing nose or over local unevenness of the wing surface. The latter mechanism of unstable wave generation may be removed by smoothing the wing surfaces. However, the leading-edge inhomogeneity is a nonremovable one. The proposed methods of boundary-layer laminarization are based on the possibility of mutual cancellation of Tollmien-Schlichting waves excited near the leading edge and over artificial unevenness on streamlined surfaces. Such cancellation is achieved by a special choice of the unevenness form. This paper is devoted to a mathematical justification of the methods in question.

Let us consider a two-dimensional flow over a straight smooth wing at low Mach number. We introduce a coordinate system with an origin at some point on the wing surface, with streamwise x-axis, and with y-axis normal to it. We denote time by t and the disturbance streamfunction by ψ. We suppose that the boundary-layer flow is laminar near the coordinate origin. Furthermore, we suppose that the boundary layer is disturbed by sound waves having a frequency ω. As stated above, the acoustic wave generates Tollmien-Schlichting waves in the vicinity of the leading edge. The disturbance of the boundary-layer flow due to this unstable wave near the coordinate origin takes the form

$$\psi_{\text{T-S}} = \epsilon\, c\, \phi(y) \exp(i\alpha x - i\omega t) + \text{C.C.}$$

Here ϵ denotes the complex amplitude of the acoustic wave, α denotes the complex wavenumber of the Tollmien-Schlichting wave, and c denotes its complex coupling coefficient; $\phi(y)$ is the complex eigenfunction. This wave is amplified as it propagates downstream, and then it induces laminar-turbulent transition.

In order to enlarge the length of the laminar flow portion, we change the local surface geometry. Let us suppose that the wing surface has a two-dimensional artificial unevenness, near the origin,

$$y_w = f(x). \tag{1}$$

The height of the unevenness (1) is assumed to be small with respect to the boundary-layer thickness. As Ruban (1984) showed, such unevenness generates Tollmien-Schlichting waves by sound scattering. The form of this 'artificial' unstable wave is

$$\psi'_{\text{T-S}} = \epsilon\, F(\alpha)\, A\, \phi(y) \exp(i\alpha x - i\omega t) + \text{C.C.}\ ,$$

where A is a specific complex coupling coefficient, and $F(\alpha)$ is the Fourier transform of the unevenness form defined by:

$$F(k) = \int_{-\infty}^{+\infty} f(x) \exp(-ikx)\, dx. \tag{2}$$

Consider the family of unevennesses

$$f = h\, g(x - X), \tag{3}$$

where h and X characterize the height and the location of the unevenness, respectively. The phase of the artificial unstable wave grows linearly with X, so the unevenness location allows control of this phase. The amplitude of the generated wave may be controlled by varying the unevenness height.

Let

$$X = \frac{\pi + arg\ G(\alpha) + arg\ A - arg\ c}{\operatorname{Re} \alpha},$$

$$h = \frac{|c|}{|G(\alpha)|\ |A|} \exp(-X \operatorname{Im} \alpha),$$

where G is the Fourier transform of the function g (see Manuilovich, 1990). As a result, the condition of Tollmien-Schlichting wave cancellation is satisfied:

$$\psi_{\text{T-S}} + \psi'_{\text{T-S}} \equiv 0. \tag{4}$$

Note that the form of the unevenness cancelling the incoming Tollmien-Schlichting wave is independent of ϵ. So the condition of cancellation (4) is fulfilled automatically regardless of the values of the amplitude and phase of the acoustic waves. The above-described method of Tollmien-Schlichting wave cancellation was suggested in Manuilovich (1990).

Thus far it has been assumed that the acoustic field is mono-harmonic. In the real situation an engine noise spectrum contains multiple frequencies. The Tollmien-Schlichting waves corresponding to these frequencies may also be amplified by an unstable boundary-layer flow. In this connection, it is necessary to develop a method for cancellation of a number of Tollmien-Schlichting waves with distinct frequencies. The following circumstances indicate the possibility of such a generalization.

First, note that the mono-harmonic receptivity problem can be generalized directly to the case of discrete spectra of sound: the proper receptivity problems may be solved independently for each value of the acoustic frequency. Each Fourier component of sound generates two unstable waves of the appropriate frequency (in the vicinity of the leading edge and over the unevenness). Second, the condition of Tollmien-Schlichting wave cancellation (4) is equivalent to the ratio

$$F(\alpha) = -\frac{c}{A}. \tag{5}$$

From the viewpoint of Fourier analysis, the equality (5) is in fact the condition imposed on the unevenness Fourier transform at the only point $k = \alpha$ (in the complex k-plane) corresponding to the sound frequency. Considering more complicated families of unevenness than (3), it is possible to satisfy the condition of cancellation for a set of frequencies. Thus the conditions of Tollmien-Schlichting wave cancellation for N frequencies present N complex equalities (5). Consider the system of $2N$ fundamental humps (or hollows) placed at constant step d,

$$f = \sum_{m=1}^{2N} c_m \, g(x - md). \tag{6}$$

Substituting (6) into the above equalities and separating the real and imaginary parts, we obtain a real system of $2N$ linear algebraic equations for the real amplitudes c_m. The value d of the step in (6) must be chosen so that the system in question would be solvable.

Introduce the total coupling coefficient $c_{\text{T-S}}$, i.e. the ratio between the complex amplitudes of the resulting Tollmien-Schlichting wave and the acoustic wave generating it by

$$c_{\text{T-S}} = c + F(\alpha)\, A.$$

This ratio characterizes the receptivity of the boundary-layer flow with respect to acoustic disturbances. At a given amplitude of sound, a decrease of $|c_{\text{T-S}}|$ leads to an increase of the length of laminar flow, so that the magnitude of the total coupling coefficient is a significant parameter which should be taken into account in laminarized airfoil design. The methods of boundary-layer laminarization described in this paper may be regarded as reducing the total coefficient to zero by means of changing the airfoil geometry.

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FASTEST GROWING GÖRTLER VORTICES IN COMPRESSIBLE BOUNDARY LAYERS

by

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The present investigation is motivated by recent interest in the development of hypersonic aircraft which might well be capable of reaching speeds of Mach number in the order of 20-25. Of concern is the importance of Görtler vortices in boundary layers of regions of curvature and, in particular, their growth rates. They may be an important factor in the transition from a laminar flow to a turbulent one.

In the incompressible case Denier, Hall, and Seddougui (1991), hereafter referred to as DHS, showed that for large Görtler numbers, the most dangerous Görtler vortices have wavelengths small compared to the boundary-layer thickness and are trapped near the wall.

For compressible flows the fastest growing Görtler vortices for O(1) Mach numbers have been identified by Dando and Seddougui (1991), hereafter referred to as DS. They showed that in the inviscid limit of large Görtler number and O(1) wavenumber, two modes exist which can be described by parallel flow effects. One mode is trapped in a layer near the edge of the boundary layer with growth rate tending to a constant as the wavenumber increases. The other mode has the vortex activity confined to a thin layer adjacent to the wall with growth rates larger than those of the trapped-layer modes and increasing as the disturbance wavenumber increases. Thus, the inviscid limit does not predict a fastest growing mode.

Inspection by DS of a viscous mode close to the right-hand branch of the neutral curve showed that the growth rate of this mode increases as the wavenumber decreases. This mode was shown to exist in a thin layer away from the wall.

As suggested by the above results, DS showed that there exists an intermediate region where the viscous mode and the inviscid wall layer mode overlap. This occurs when their growth rates are the same size. In this intermediate region the vortices are confined to a thin wall layer and are governed by viscous effects. This situation for O(1) Mach numbers is the same as that for the corresponding incompressible case since the effects of compressibility may be scaled out leaving the incompressible problem solved by DHS. It was shown by DHS that the most unstable mode occurs in this intermediate region.

The significance of this most unstable mode, apart from having a larger spatial growth rate than the inviscid wall layer modes or the viscous mode close to the right-hand branch of the neutral curve, is that it occurs close to the wall. This suggests that significant coupling coefficients will be possible in the receptivity problem for the most unstable modes. This was shown to be the case for the incompressible problem by DHS, and identical results for a compressible fluid with O(1) Mach number may be inferred simply from the results of DHS.

For hypersonic speeds, DS showed that the inviscid trapped layer mode is located in the logarithmically small temperature adjustment layer at the edge of the boundary layer. Of concern is whether the most dangerous mode for hypersonic speeds occurs close to the wall, as in the incompressible and supersonic cases, or at the edge of the boundary layer in the temperature adjustment layer, as this will have important consequences in the receptivity problem as described above. By considering the large Mach number limit of the most dangerous supersonic mode described above, we show that the growth rate of this wall layer mode decreases as the Mach number increases. This suggests that the wall layer mode may not be the most important mode for hypersonic flow. We show that this is indeed the case.

Two modes, in addition to that described by DS, have been identified trapped in the temperature adjustment layer. The first by Hall and Fu (1989) who described a viscous mode governed by parallel effects, and the second by Fu and Hall (1992) who considered crossflow effects on an inviscid mode. We show that each of the hypersonic modes described above may be the fastest growth mode, depending on the size of the Görtler number.

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ASYMPTOTIC THEORY OF GORTLER VORTICES

by

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The occurrence and development of Gortler vortices in a boundary layer near a concave surface is investigated for high Reynolds number $Re_\infty = U_\infty L/\nu = \epsilon^{-2} >> 1$ and Gortler number $G_\infty = 2Re_\infty^{1/2} L/R = 2K\text{æ}/\epsilon >> 1$. Here U_∞ is the uniform free-stream velocity, L is the distance along the surface from its leading edge to the vortex incipience point, ν is the kinematic viscosity, R is the surface curvature radius, and $L/R = \text{æ}K$, $K \sim 1$, $\epsilon < \text{æ} < 1$. Using the method of matched asymptotic expansions (Van Dyke, 1964), a solution of the Navier-Stokes equations is constructed for three-dimensional disturbed regions with characteristic dimensions Δx, Δy and Δz; the x-axis is the flow direction, the y-axis is normal to the surface and the z-axis is in the transverse direction.

It is found that for $\Delta y \sim \Delta z \sim \epsilon^{6/5}/\text{æ}^{1/5} < \delta \sim \epsilon$, $\Delta x \sim (\epsilon/\text{æ})^{3/5} < 2$ (δ is the characteristic boundary layer thickness), the incipience and development of vortices closely adjacent to the surface is described by the solution of the following boundary-value problem:

$$u_x + v_Y + w_z = 0, \quad Re_1(uu_x + vu_Y + wu_z) = u_{YY} + u_{zz} \ , \tag{1}$$

$$Re_1(uv_x + vv_Y + wv_z + u^2 + p_Y) = v_{YY} + v_{zz} \ ,$$

$$Re_1(uw_x + vw_Y + ww_z + p_z) = w_{YY} + w_{zz} \ ,$$

$$u = v = w = 0 \ (y = 0); \quad Re_1 = AK^{1/2}(\lambda/2\pi)^{5/2} \sim 1 \ ,$$

$$u \to y, \ v, w \to 0, \ p \to -y^3/3 \quad (\text{as } x \to -\infty \text{ or } y \to \infty)$$

$$u, v, w, p(x,y,z) = u, v, w, p(x,y,z+2\pi); \quad A = (u_{0Y})_{Y=0},$$

where u, v, w are the velocity components, p is the pressure, Re_1 is the local Reynolds number, λ is the vortex wavelength and $u_o(y)$ is the velocity profile in the boundary layer at the point of vortex incipience. Numerical solutions for linear theory (Van Dyke, 1964)

$$\Delta f(x,y,z) = F(y) \exp(\beta x)(\sin z, \cos z), \ f = u, v, w, p \tag{2}$$

have been obtained by Bogolepov et al. (1988) and Timoshin (1990). It was found that the reduced vortex amplitude increment $\sim \beta/Re_1^{1/5}$ is maximum. As $Re_1 \to \infty$, the dissipative terms (Van Dyke, 1964) vanish, and the solution for linear theory (Bogolepov, 1988) was obtained analytically

$$\beta_n = n^{-1/2}, \ n = 1, 2, 3, \ldots \tag{3}$$

This agrees with the numerical solutions (Timoshin, 1990; Bogolepov and Lipatov, 1992). As $Re_1 \to 0$, $\beta_n \sim Re_1$ and convective terms also vanish (Van Dyke, 1964); but then the convection mechanism required for vortex generation is excluded from the consideration. Therefore, an increase in λ must be compensated for by a rise in characteristic velocity, so that the local Reynolds number remains finite; this is possible when the vortices move away from the surface (Hall, 1982; Bogolepov and Lipatov, 1992).

For the vortices lifted by a characteristic height $h \sim æ\Delta x^2 \leq \delta \sim \epsilon$, $(\epsilon/æ)^{3/5} < \Delta x < (\epsilon/æ)^{1/2} < 1$ with $\Delta y \sim \epsilon \Delta x^{1/3}$ and $\Delta x \sim (\epsilon^3/æ\Delta x)^{1/2}$, the boundary-value problem is as follows:

$$v_Y + w_Z = 0, \quad u_X + v(1+u_Y) + wu_Z = u_{ZZ}, \quad v_X + vv_Y + wv_Z + G_1 u = v_{ZZ} \ , \tag{4}$$

$$w_X + vw_Y + ww_Z + p_Z = w_{ZZ}, \quad u, v, w, p \rightarrow 0 \quad (x \rightarrow -\infty \text{ or } y \rightarrow \pm\infty) \ ,$$

$$u, v, w, p(x, y, z) = u, v, w, p(x, y, z+2\pi), \quad G_1 = 2Ku_0 u_{0Y}(\lambda/2\pi)^4 \sim 1 \ ,$$

where u, p are the disturbance functions and G_1 is the local Gortler number. The solution of the system (4) in a linear approximation (2) is derived analytically and

$$G_1 = (1+\beta_n)^2, \quad n = 1, 2, 3, \ldots$$

This determines the value $G_1 = 1$ for neutral short-wave vortices. The solutions (1) and (4) are matched as $Re_1 \rightarrow 0$ or $h \rightarrow \epsilon^{6/5}/æ^{1/5}$ (Bogolepov and Lipatov, 1992).

When the vortices occupy the entire width of the boundary layer, i.e. with $\Delta x \sim (\epsilon/æ)^{1/2} < 1$, $\Delta y \sim \Delta z \sim \delta \sim \epsilon$, their incipience and development is described by the solution of the following boundary-value problem

$$u_X + v_Y + \gamma_3 w_Z = 0, \quad uu_X + vu_Y + \gamma_3 wu_Z = 0 \ , \tag{5}$$

$$\gamma_2\gamma_3(uv_X + vv_Y + \gamma_3 wv_Z) + u^2 + p_Y = 0 \ ,$$

$$uw_X + vw_Y + \gamma_3 ww_Z + p_Z = 0, \quad v=0 \ (y=0) \ ,$$

$$u \rightarrow u_o(\gamma_1 y), \quad p \rightarrow -\int_0^Y u_0^2 dy, \ v, w \rightarrow 0 \quad (x \rightarrow -\infty \ \text{ or } \ y \rightarrow \infty) \ ,$$

$$u, v, w, p(x, y, z) = u, v, w, p(x, y, z+2\pi).$$

For simplicity, a velocity profile corresponding to an intensive suction is used in the boundary layer according to

$$u_0 = 1 - \exp(-\gamma_2 y), \quad v_0 = -2/\delta_1, \quad u_0(\delta_1) = 0,99 \ , \tag{6}$$

where δ_1 is constant. If the vortices are located inside the boundary layer and $\lambda \leq \delta_1$, then $\gamma_1 = \lambda/2\pi\delta_1$, $0 < \gamma_1 \leq 1$, $\gamma_2 = \gamma_3 = 1$ and the linear solution of equations (5) and (6) are obtained in the analytical form, viz.

$$\beta_n^{-2} = \gamma_1(n^2-1)/2 + n, \quad n = 1, 2, 3, \ldots$$

which reduces to equation (3) with $\gamma_1 \rightarrow 0$. If $\lambda \geq \delta_1$, then $\gamma_1 = \gamma_3 = 1$, $\gamma_2 = (2\pi\delta_1/\lambda)^2$, $0 < \gamma_2 \leq 1$ and

$$\beta_n^{-2} = (n^2-1)/2 + \gamma_2^{1/4} n, \quad n = 1, 2, 3, \ldots. \tag{7}$$

It should be noted that $\beta_1 = \gamma_2^{-1/4} \rightarrow \infty$ as $\gamma_2 \rightarrow 0$, which means that the incipience of the first vortex mode takes place at smaller distances than that of all subsequent modes.

If the vortices are not localized inside the boundary layer and $\lambda \geq \delta_1$, then $\gamma_1 = \gamma_2 = 1$, $\gamma_3 = 2\pi\delta_1/\lambda$, $0 < \gamma_3 \leq 1$ and

$$\gamma_3/\beta_n^2 = (n^2-1)/2 + \gamma_3 n, \quad n = 1, 2, 3, \ldots . \tag{8}$$

Here only $\beta_1 \to 1$ as $\gamma_3 \to 0$, and $\beta_n \sim \gamma_3^{1/2} \to 0$ for $n > 1$. In the general case, with $\Delta x \sim (\epsilon/æ)^{3/7} < 1$, $\Delta y \sim \Delta z \sim æ^{1/7}\epsilon^{6/7} > \delta \sim \epsilon$, the triple-deck structure of a disturbed flow (Hall, 1983; Roshko et al., 1988; Timoshin, 1990; Bogolepov and Lipatov, 1992) is realized, and in the linear approximation (2)

$$\gamma_5\beta^2 - 3\gamma_4\beta^{5/3}Ai'(0) = 1 \tag{9}$$

is obtained, where γ_4 and γ_5 represent the extents of layer interaction between each other. When $\gamma_3 = \gamma_4 = 0$ and $\gamma_5 = 1$, it follows from (8) and (9) that $\beta_1 = 1$.

For long-wave vortices with $\Delta x \sim 1$, $\Delta y \sim \epsilon \sim \delta$, $\Delta z \sim (æ\epsilon)^{1/2} > \delta \sim \epsilon$, we have a boundary-value problem of the following form (see also Hall, 1983, 1988).

$$\begin{aligned}
&u_X + v_Y + w_Z = 0, \quad Re_2(uu_X + vu_Y + wu_Z) = u_{YY} \ , \qquad (10)\\
&u^2 + p_Y = 0, \quad Re_2(uw_X + vw_Y + ww_Z + p_2) = w_{YY} \ ,\\
&u = w = 0,\ v = (\delta_1/Re_2)v_{0u}\ (y=0), \quad u \to 1,\ w \to 0,\ p \to -\int_Y^{Y} u^2 dy \ \ (y \to \infty) \ ,\\
&u = u_0,\ \ v = (\delta_1/Re_2)v_0, \quad w = 0, \quad p = -\int_0^{Y} u_0^2\, dy \ \ (x = (Re_2/\delta_1^2)x_0) \ ,\\
&u, v, w, p(x, y, z) = u, v, w, p(x, y, z+2\pi) \quad Re_2 = 2\pi K^{1/2}\delta_1^{5/2}/\lambda \sim 1,
\end{aligned}$$

where $x_0 \sim 1$ is the streamwise coordinate corresponding to vortex incipience, and Re_2 is the local Reynolds number. It is worth noting that it is only necessary to take into account longitudinal variations of the flow functions in this regime. The numerical solutions of the systems (6), (10) for linear theory show that $\beta_1 = 0$, when $Re_2 \approx 0486$ and $\beta_1 \to (-3Ai'(0))^{-3/5} \approx 1.165$, as $Re_2 \to \infty$ (see (9) at $\gamma_4 = 1$ and $\gamma_5 = 0$) and $\beta_2 = 0$ with $Re_2 \approx 2.32$ and $\beta_2 \to (2/3)^{1/2} \approx 0.816$ when $Re_2 \to \infty$ (see (7) with $\gamma_2 \to 0$).

In summary, a classification scheme of vortex incipience and development has been constructed. The numerical solution of the system (1) is obtained in a one-mode approximation. A considerable reduction in the rate of the vortex amplitude increase and a significant distortion of the initial velocity profile in the boundary layer are predicted; these are caused by nonlinear interaction of the flow field disturbances. A variety of velocity component disturbance profiles are presented.

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OBLIQUE WAVES INTERACTING WITH A NONLINEAR PLANE WAVE

by

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The downstream evolution of a resonant triad of initially noninteracting linear instability wave in a boundary layer with a weak adverse pressure gradient is considered. The triad consists of a two-dimensional fundamental mode and a pair of equal-amplitude oblique subharmonic modes that form a standing wave in the spanwise direction. The growth rates are small, and there is a well-defined common critical layer for these waves. The flow outside the critical layer remains a linear perturbation about the steady two-dimensional boundary-layer flow and is described by small-growth-rate solutions to the Rayleigh stability problem, but the critical-layer flow evolves through a number of different stages. As in Goldstein and Lee (1992), the wave interaction takes place entirely within this critical layer and is initially of the parametric-resonance type which enhances the spatial growth rates of the subharmonic but leaves that of the fundamental unaffected. In contrast to Goldstein and Lee (1992), where the plane wave is completely linear in the parametric-resonance stage, the initial subharmonic amplitude is assumed small enough so that the fundamental undergoes nonlinear saturation due to self-interaction effects within its own critical layer before it is affected by the subharmonic. The initial wave interaction is weak in the sense that it enters the critical-layer problem that produces the subharmonic velocity jump as an inhomogeneous term determined at lower order (rather than through a coefficient). The two-dimensional fundamental mode exhibits linear growth in this stage and the subharmonic amplitude is explicitly determined by a single integro-differential equation. The downsteam asymptotic expansion of the analytic solution to this equation determines the scaling for the next stage of evolution in which the fundamental becomes nonlinear. The fundamental is then governed by the strongly nonlinear critical-layer problem analyzed in Goldstein, Durbin and Leib (1987), but with viscosity accounted for in the critical-layer dynamics. The subharmonic evolution is now dominated by the parametric-resonance effects and occurs on a much shorter streamwise scale than that of the fundamental. Its critical layer is therefore much thicker than that of the fundamental in this stage. The solution to the relevant subharmonic-amplitude equation is the downstream asymptotic expansion of the solution for the previous stage, but with the linearly growing fundamental amplitude replaced by the corresponding strongly nonlinear critical-layer solution. The subharmonic amplitude continues to increase during this parametric-resonance dominated stage, even when the fundamental amplitude saturates, and it eventually becomes large enough to influence the fundamental. This leads to a new stage of development in which all waves evolve on the same shorter streamwise length scale, which means that the fundamental and subharmonic critical layers are again of equal thickness. The relevant amplitude equations are then the same as in Goldstein and Lee (1992), but with the linear growth terms omitted.

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THEORY OF THREE-DIMENSIONAL HYPERSONIC FLOW OVER A WING AT MODERATE ANGLES OF ATTACK

by

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This study is devoted to the development of an asymptotic approach in the theory of three-dimensional hypersonic flow over a thin wing at finite angle of attack. This flow regime is characterized by a dominant contribution of the lower-wing surface to aerothermodynamic characteristics. A thin layer of gas, compressed by a strong bow shock wave, covers the lower surface. Additional compression is due to intensive thermochemical reactions (primarily, high temperature disassociation), which results in a decrease of the effective ratio of specific heats. Therefore, the well-known thin shock layer concept (G. G. Chernyi, W. D. Hayes, J. D. Cole) as a limit (corresponding to the tendency of specific heat ratio γ to unity) has proved to be very productive for both development of the full theory and practical calculations having acceptable accuracy. In the asymptotic thin shock layer method, the limiting process $\gamma \to 1$, $M_\infty \to \infty$ is used, and the solution is sought in the form of expansions in a small parameter ϵ, which characterizes the inverse density ratio across the strong bow shock. The limiting case of an infinitely thin shock layer ($\epsilon = 0$) corresponds to a Newtonian flow model. In the present study, the theory of the first approximation described a three-dimensional thin shock layer structure on a wing of small aspect ratio. The most general case is studied when the wing aspect ratio and thickness have the same order of magnitude as the Mach angle and the layer thickness respectively (A. F. Messiter). In this case, the bow shock is attached to the wing apex, but can be either attached to or detached from a sharp leading edge. Also the leading term expansion for the shock shape is unknown a priori and Rankine-Hugoniot relations are applied on a shock surface, which itself must be determined from the solution. According to hypersonic cross section law for slender bodies at high angles of attack (V. V. Sychev), an approximate similarity law is obtained, which correlates numerical and experimental data over a fairly broad range of parameters. A new integral of gas motion in a thin shock layer was discovered. It has shown conservation of the streamwise vorticity component along streamlines. This made it possible to obtain the general analytic solution of the nonlinear shock layer equations. Thus the initial three-dimensional problem reduces to quadratures for the gas dynamic functions and to a simpler two-dimensional integro-differential equation, relating the shock and the wing shapes. Analytical and numerical solutions for delta wings, as well as for more complex wing planforms, have shown interesting flow properties as follows: (i) stabilization of shock shape in a conically-transonic region, (ii) local pressure peaks in the vicinity of the symmetry plane, (iii) formation of additional lines of flow convergence and divergence, and (iv) intensive vorticity generation near the point of shock detachment from a curvilinear leading edge. Analysis of some singularities was accompanied by separation and asymptotic consideration of corresponding subregions in the flow field, for example, the neighborhoods of singular cross section with multi-zone structure Comparisons with calculated numerical results and experimental data show that the first approximation considered yields Newtonian local and total aerodynamic characteristics if $\epsilon < 1$, even though the asymptotic theory requires that $\epsilon \ll 1$.

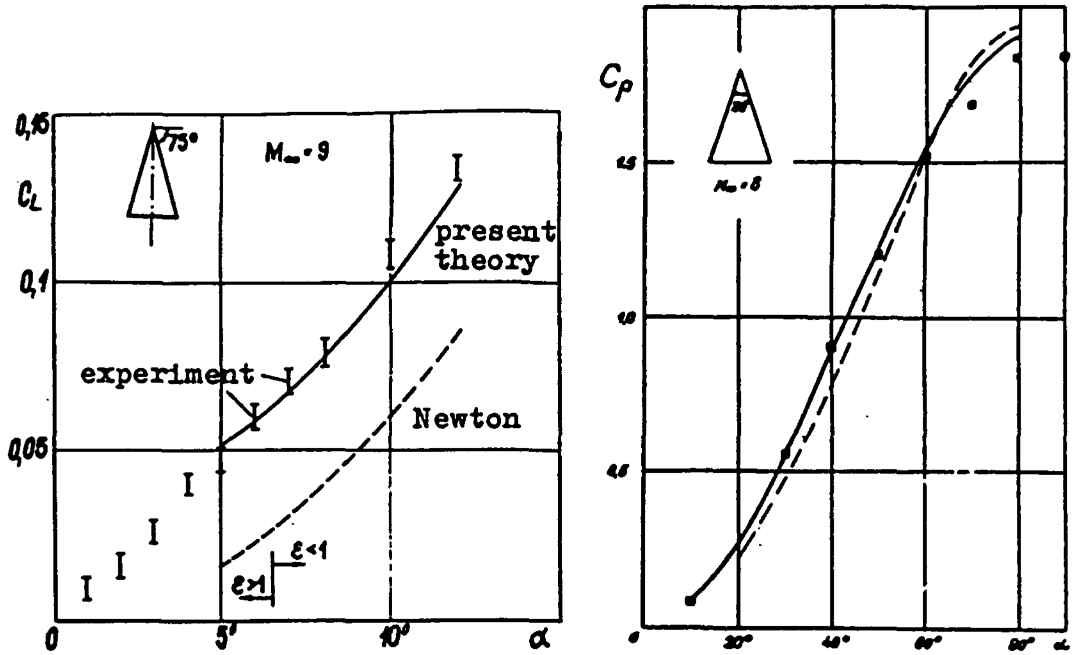

Figure 1. Lift and centerline pressure coefficient vs. incidence.

The application of present analytical solution has allowed the formulation and solution of a variational problem to determine an optimal wing shape having a maximum hypersonic lift-to-drag L/D in the first approximation as compared to a Newtonian one. Finally, the problem is reduced to minimization of a linear functional under different constraints. An explicitly analytical form of the functional shows that to produce sufficiently large L/D, the base plane projection of the leading edge of the wing must lie below the attached shock section by this plane passing through a straight trailing edge. As a result, the optimized wing of a given planform area and span has a concave lower surface and a forward part which is not bent down, at least near the leading edges. A bifurcational behavior in the optimization process has been found with an abrupt switch from a sharp apex planform to a cut apex, if the span is reduced (see Figure 2).

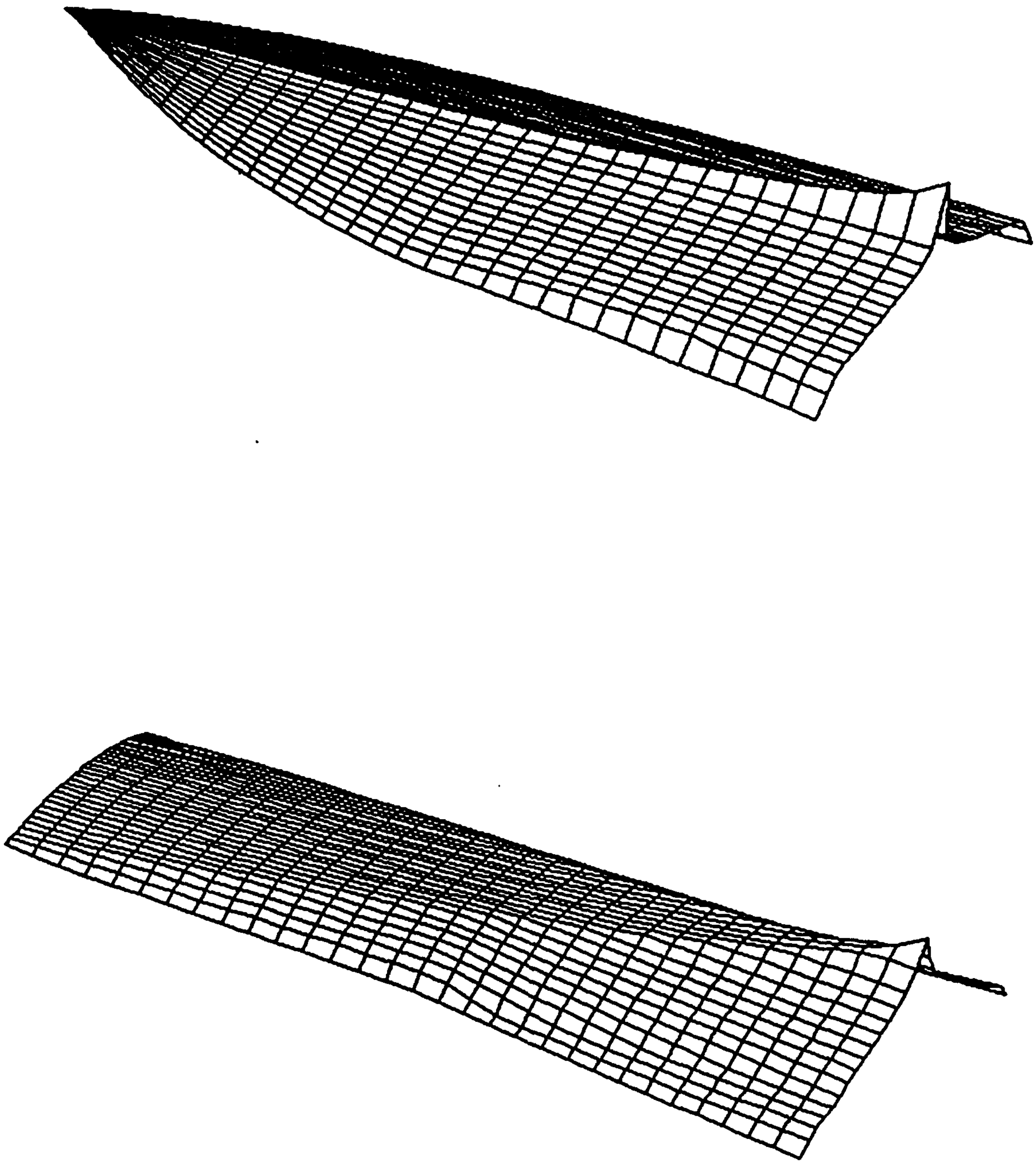

Figure 2. Optimized hypersonic wings.

THE INFLUENCE OF SURFACE COOLING ON COMPRESSIBLE BOUNDARY-LAYER STABILITY

by

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To prevent damage to the surface of aircraft during supersonic and hypersonic flight, surface cooling is often applied, especially at high Mach numbers. It is therefore of interest to know the effects of such cooling on the instability and transition properties of the local compressible boundary layer. This investigation discusses theoretically the influence of surface cooling on compressible boundary-layer stability at high Reynolds numbers, Re, with special regard to the high Mach number limit.

Surface cooling enhances the heat transfer and shear stress of the basic boundary-layer flow at the surface. The increased surface velocity gradients cause the typical viscous-inviscid wavelength to decrease and so alter the three-tier, triple-deck flow structure associated with the TS wave. For large values of the free-stream Mach number M_∞, the mean flow has a two-tier form to allow the temperature to adjust from being $O(M_\infty^2)$ near the wall to being unity in the free stream. This has the effect of increasing the wavelength of the viscous modes. Indeed when M_∞ is $O(Re^{1/10})$, we have to account for the non-parallelism of the basic flow. Thus the combination of high Mach number and surface-cooling is a delicate balance.

The first flow structure we shall consider is that for the moderate cooling, where the surface temperature T_w is $O(Re^{-1/12})$ and M_∞ is $O(1)$. This of the compressible Rayleigh inviscid type across the majority of the boundary layer, but is quasi-steady in form, which admits a pressure-displacement interaction with the viscous sublayer. More cooling increases the spatial growth rates still further, with the disturbance being concentrated relatively near the surface and the flow structure again changes. The resulting severe cooling flow structure allows compressibility effects to enter the dynamics of the thin viscous sublayer. This work is an extension of that by Seddoughi et al. (1990), who showed that for the linear Chapman law the spatial growth rates become comparable with the growth rates of inviscid modes, rendering previously stable modes unstable. Here we apply the nonlinear viscosity-temperature ($\mu \propto T^\alpha$) law which is, especially at high Mach number, more physically realistic. Further we also discuss the transonic realm, cross-flow effects, and consider upper-branch properties.

The second study is the effect of wall cooling on the viscous-inviscid TS modes when the outer flow is hypersonic. The present analysis addresses the weak interaction regime ($\chi = M_\infty^3 Re^{-1/2} \ll 1$). For a surface temperature

$$T_w = O(\chi^{\frac{1}{4\alpha = 2}}),$$

the flow structure is of triple-deck type with a length-scale $O(\chi)$. The neglect of any pressure variation across the main deck requires

$$Re^{\frac{1}{2(8\alpha+7)}} \ll M_\infty \ll Re^{1/6},$$

the lower limit corresponding to moderate cooling at large Mach number. Further cooling leads to equations identical in form to the case where χ is O(1) and the 'Newtonian approximation' of specific heat close to unity applies.

Given the increased-instability properties found here for cooled surfaces, the corresponding nonlinear processes at higher disturbance amplitudes should also be of much interest with respect to transition. We hope to consider both the finite time nonlinear break-up and forms of vortex-wave interaction. For example, the high-frequency form of the moderate-cooled stage suggests a possible avenue of attack.

A LINEAR PROBLEM OF A VIBRATION IN A BOUNDARY LAYER ON A PARTIALLY ELASTIC SURFACE

by

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Within the theory of the plane-parallel boundary layer with free interaction, the linear problem of a vibrator on a flat plate is considered. Usually, such a problem is studied for a stiff plate. In this work, a part of the plate behind the vibrator is assumed to be elastic.

Usually the normal tension on the surface of the plate is greater than the tangential one and the model of elastic interaction between gas and surface is chosen. It means that the deviation of the elastic part from its neutral state is proportional to the excess pressure:

$$-p = \kappa_o y_w, \qquad c < x < d,$$

where p is the excess pressure, κ_o is the stiffness coefficient, y_w is the deviation of the elastic surface from the neutral position $y = 0$. Whereas the application of Laplace and Fourier transforms in a problem with a completely stiff plate leads to an explicit expression from the surplus pressure image, the problem with a partially elastic plate is reduced to an integral equation for the pressure on the elastic part of the plate,

$$p(t, x) = \frac{1}{4\pi^2 i} \int_{-\infty}^{\infty} e^{i\kappa x}\, d\kappa \int_{\ell - i\infty}^{\ell + i\infty} e^{\omega t} \frac{|\kappa|\, A_i'(\Omega)}{A_i'(\Omega) - (i\kappa)^{1/3} |\kappa| \int_{\Omega}^{\infty} A_i(z)\, dz} \times$$

$$\left[-\bar{f}_o(\omega, \kappa) + \int_0^{\infty} e^{-\omega t_1}\, dt_1 \int_c^d \frac{e^{-i\kappa x_1} p(t_1, x_1)}{\kappa_o}\, dx_1 \right] d\omega\,,$$

$$\Omega = \omega / (i\kappa)^{2/3},$$

where $\bar{f}_o(\omega,\kappa)$ is the Laplace−Fourier image of the vibrator $f_o(t, x)$ and $A_i(z)$ is the Airy function. After solving this equation, the determination of the whole pressure is done with traditional methods of computation of the inverse Laplace-Fourier transforms. It is found that in such a formulation of the unsteady problem, for the start of a harmonic vibrator ($f_0(t, x) = 0$ for $t < 0$; $f_0(t, x) = f(x) \sin \omega_0 t$ for $t \geq 0$), the excess pressure tends to a harmonic oscillation with a limiting amplitude at large time. This means that in linear problem there is no resonance leading to an unbounded pressure increase. The parameters of this oscillating regime depend on the vibrator frequency and the length of the elastic part. By changing the stiffness coefficient and the length of the elastic part, it is possible to affect the Tollmien-Schlichting waves generated by the vibrator. It is found that for some conditions, the elastic part can decrease the amplitude of the Tollmien-Schlichting waves.

VORTEX-WAVE INTERACTION IN A STRONG ADVERSE PRESSURE GRADIENT

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It is well-known that in the presence of a large adverse pressure gradient driving the high-Reynolds-number flow past a solid surface, the steady two-dimensional boundary-layer equations develop a singularity at the streamwise location at which the skin-friction vanishes (Goldstein, 1948). This result is interpreted usually as an indication that the physics involved at this stage in the flow development can no longer simply be described by the concept of a non-interactive boundary-layer driven by an external flow. The study of Stewartson (1970) indicates that this singularity is not removable in the sense that it cannot be smoothed out on a shorter length scale around the point of separation.

The present theoretical, high-Reynolds-number study considers the stability of the flow slightly upstream of separation where we introduce three-dimensional Tollmein-Schlichting waves of sufficiently large amplitude to induce strong three-dimensionality into the mean flow. The non-linear interaction of the wave with the mean flow is described in the context of triple-deck theory, leading to governing equations of the form described in Hall and Smith (1991). The assumption of a large adverse pressure gradient simplifies the boundary-layer equations considerably, and the problem reduces to one of solving for the three-dimensional skin-friction field subject to non-linear forcing by the wave. The initial development of this interaction can be studied analytically by means of a linear analysis.

It is not immediately clear as to what the ultimate effects of the wave forcing upon the skin-friction will be and, in particular, whether the flow will still separate, but subsequent numerical solutions of the governing equations indicate that the wave amplitude grows in a singular fashion as some streamwise location is approached. This position is dependent upon the spanwise wavelength imposed on the waves but is always upstream of the point at which the Goldstein singularity develops in the undisturbed flow. A feature of this 'blow-up' is that the skin-friction remains regular and positive although strong variations in the spanwise direction are observed in the numerical computations. To study the interaction further will require the consideration of the flow development over a shorter streamwise length scale with the pressure-displacement interaction between the boundary-layer flow and the induced free-stream response becoming significant. At this stage, it is unclear what the result of this new effect will be.

In summary, it would seem possible, at least in principle, that the separation of the boundary-layer could be delayed by the introduction of three-dimensional disturbances of the appropriate amplitude, slightly upstream of the expected separation point.

A similar local stability analysis close to the separation point is possible for mean flows of a marginally-separating nature where the skin-friction tends to zero in a regular fashion. Once again, numerical integration of the non-linear governing equations indicates the possibility of a finite distance blow-up in the wave amplitude, with the location of the singular point now dependent upon the angle of inclination to the free-stream as well as the spanwise periodicity of the wave.

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WEAK TURBULENCE IN PERIODIC COMPRESSIBLE FLOWS

by

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In spite of numerous investigations of hydrodynamic stability, it remains difficult to obtain analytical results even for incompressible flows and even in the context of linear stability theory. In this study we find the exact value of the critical Reynolds number R_C for a class of special viscous gas flows, namely two-dimensional flows, induced by a unidirectional external force field that is periodic in one of the spatial coordinates. It is known that the simplest such flow, the so-called Kolmogorov flow, permits a thorough study of the stability problem for incompressible fluids, and it is considered as an example through which transition to turbulence may be studied. As far as we know, this approach has not been completely fulfilled even in the two-dimensional incompressible case and even numerically. We perform the first part of the research for the compressible case, i.e., we predict the long-wave instability when the Reynolds number exceeds some threshold value and describe the formation of a large-scale streamwise coherent structure for Kolmogorov-like flows. For the rather simple case of a monatomic gas with constant transport coefficients the Kolmogorov flow (in the dimensionless form) becomes (Brutyan and Krapivsky, 1992)]:

$$u = \sin(y), \quad T = p_\infty + \frac{\eta}{2\varkappa}\cos^2(y), \quad p = p_\infty, \quad \rho = p/T \tag{1}$$

with $p_\infty = 3/(5M^2)$ where M is the Mach number.

Let us assume the flow (1) becomes unstable at some critical Reynolds number R_C. The determination of R_C is based on the assumption that near the stability threshold $0 \leq R - R_C \ll 1$, the critical wave number approaches zero. This suggests that a long wavelength approximation can be applied to the Navier-Stokes equations. In order to use the long-wave approximation, a small parameter ϵ is introduced by the relationship $R^{-1} = (1-\epsilon^2)R_C^{-1}$. The introduction of ϵ is motivated by the previous studies (see, e.g., Meshalkin and Sinai, 1961; Nepomnyashchy, 1976; Sivashinsky, 1985; and Brutyan and Krapivskky, 1991) of the stability of the Kolmogorov-like flows in an incompressible fluid, where it was found that the critical wave number for the onset of instability is proportional to ϵ. Thus the typical size of a coherent vortex in the longitudinal direction is much greater than the period of the flow ($x \simeq \epsilon^{-1}$).

As in the incompressible case, we assume that the onset of instability for the Kolmogorov flow in a viscous compressible gas is also associated with zero wave number. Hence we shall take advantage of a long-wave character of the instability. An analogy with the incompressible case (Meshalkin and Sinai, 1961; Nepomnyashchy, 1976; Sivashinsky, 1985; and Brutyan and Krapivskky, 1991) suggests introducing slow space-time variables and rescaling the hydrodynamic variables according to

$$\hat{x} = \epsilon x, \quad \hat{y} = y, \quad \hat{t} = \epsilon^4 t, \quad u = \hat{u}, \quad v = \epsilon\hat{v}, \quad T = \hat{T}, \quad p = p_\infty + \epsilon\hat{p}, \quad \rho = \hat{\rho} \tag{2}$$

Equations (2) indicate that near the stability threshold a large scale longitudinal coherent structure is formed. We see that the streamwise size of this structure scales as $(R - R_C)^{-1/2}$, while the temporal scale is $(R - R_C)^{-2}$.

Substituting (2) into the Navier-Stokes equations and then expanding the kinematic and thermodynamic variables, $\hat{u} = u_o + \epsilon u_1 + \ldots$ etc., we obtain a set of equations in the zeroth, first, and second approximations. In the zeroth approximation, the longitudinal velocity, density, temperature and entropy remain undisturbed, while v_o and p_o become

$$v_o = T_o \Phi(x,t), \qquad p_o = -\frac{2}{3R_C} \frac{\eta}{\mathcal{K}} \Phi \sin(2y), \tag{3}$$

where $\Phi(x, t)$ is an arbitrary function. It is worth noting that the solution (3) exists at *arbitrary* R_C. Similar analysis of the first approximation shows that the solution exists only at the following definite value of the critical Reynolds number

$$R_C^2 = \frac{p_\infty + (\eta/4\mathcal{K}) - (\eta/3\mathcal{K}) < \sin^2(2y)/T_o >}{p_\infty^2 < \cos^2(y)/T_o > + \frac{\eta}{16\mathcal{K}}\left(\frac{5\eta}{2\mathcal{K}} + 3\right) p_\infty < \sin^2(2y)/T_o >}. \tag{4}$$

Here we have used the shorthand notation $<(\ldots)> = (2\pi)^{-1} \int^{2\pi} (\ldots)dy$ to denote an average in y. For the most interesting case of a small Mach number M, formula (4) reduces to

$$R_C = \sqrt{2}\left[1 + \frac{19}{216} M^2 - \frac{485}{7938} M^4 + \ldots\right], \tag{5}$$

where the ratio $\eta/\mathcal{K}$ was approximated by 4/15 as predicted by the kinetic theory of diluted monoatomic gases. As the Mach number decreases, this result reduces to the well-known result of Meshalkin-Sinai (1961), $R_C = \sqrt{2}$.

Figure 1 represents $R_C = R_C(M)$ for $\eta/\mathcal{K} = 4/15$. The stability threshold is seen to increase with M over the interval $0 < M < M_1 (M_1 = 0.8752\ldots, R_C(M_1) = 1.4605\ldots)$ and to decrease with M over the interval $M_1 < M < M_2$ $(M_2 = 2.0222\ldots)$. The unexpected behavior of $R_C = R_C(M)$ at $M \simeq 1$ requires some explanation. A full discussion of this point will not be given here, but it may be argued that the non-monotonic behavior of $R_C = R_C(M)$ takes place outside the realm of the continuum description. Actually, a well-known relation between Mach, Reynolds and Knudsen numbers, $Kn = M/R$, shows that when $M \simeq 1$ and $R \simeq 1$, the Knudsen number is also $O(1)$, i.e., the main assumption of a continuum model is violated. Thus, the behavior of $R_C(M)$ at $M \simeq 1$ is an artifact of using the continuum Navier-Stokes equations.

The multiple-scale technique described in this paper can be generalized on arbitrary smooth periodic unidirectional gas flows $U = f(y)$. In particular for the small Mach number, the critical Reynolds number may be expressed as

$$R_C = \left(< f_1^2 >\right)^{-1/2}\left[1 + \frac{12 < f_1^2 > - 1g < f^2 f_1^2 > - 5 < f^2 > < f_1^2 >}{54 < f_1^2 >} M^2 + \ldots\right], \tag{6}$$

where f_1 is defined from the equations $df_1/dy = f$ and $< f_1 > = 0$. For the simplest Kolmogorov flow, $f(y) = \sin(y)$, equation (6) coincides with (5).

We believe that our results provide some insight into the phenomenon of spontaneous formation of large-scale coherent structures in two-dimensional turbulent compressible flows.

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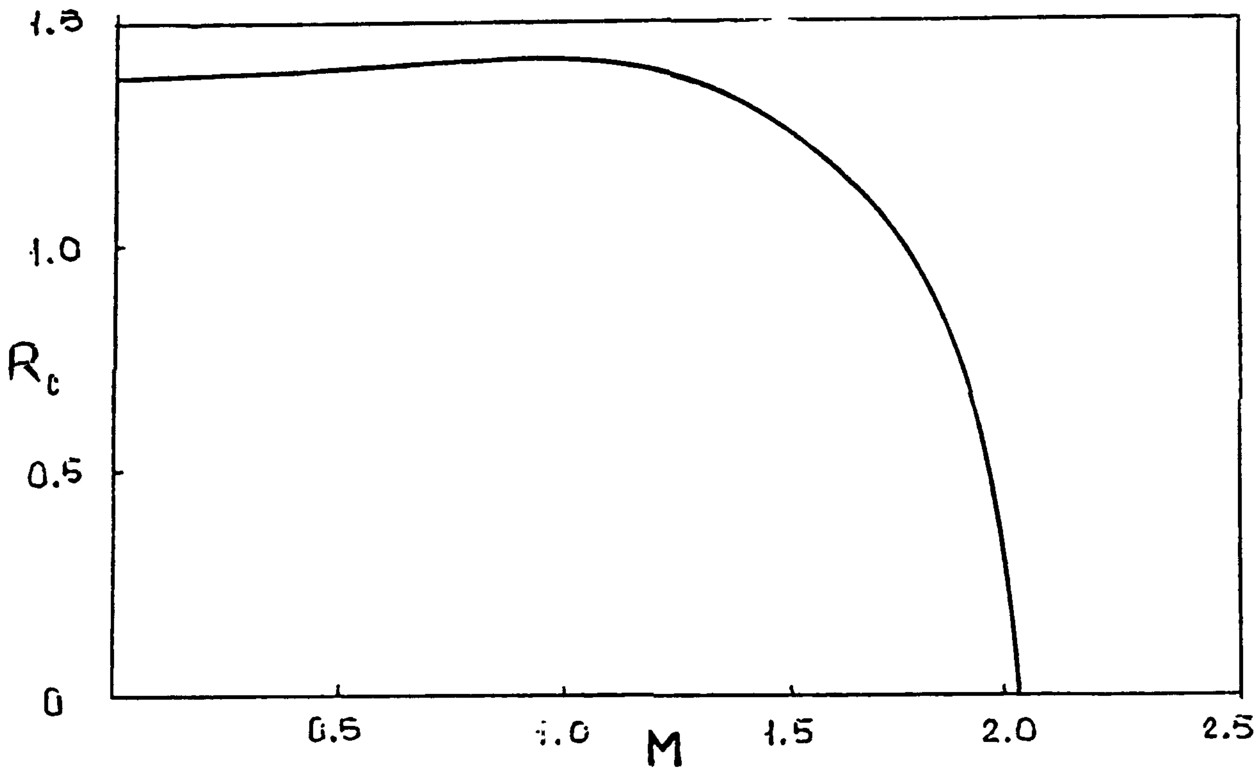

Figure 1

ON EXTENSION OF CONTINUUM MODELS TO RAREFIED GAS DYNAMICS

by

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As advances progress in the modeling of aerodynamic flows, recent works in extending the continuum gas dynamic models to rarefied hypersonic flows are based on Burnett's Equations (Fiscko and Chapman, 1988; Zong et al., 1991a; Zong et al., 1991b) and on Grad's thirteen-moment equations (Cheng et al., 1989; Cheng, 1991; Cheng et al., 1992). The needs of compatible boundary conditions in the application of these equations, the gas-kinetic basis of these models, and other theoretical and computational issues are examined along with solution examples and their comparison with Direct Simulation Monte-Carlo calculations. Prospects and limitation of their further extension to model high-temperature nonequilibrium flow for a diatomic gas far from translational equilibrium are discussed.

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SOLITON DISTURBANCES IN A TRANSONIC BOUNDARY LAYER

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We consider a compressible viscous fluid flowing past a flat plate. It is supposed that the Mach number M_∞ of the undisturbed stream is limited by the requirement $M_\infty^2 - 1 = \delta K_\infty$, where δ is small parameter and $K_\infty = O(1)$. We are interested in the neighborhood of some point on the plate at a distance L^* from the leading edge. A suitable definition of the Reynolds number is based on L^*. In the double limit $Re \to \infty$, $\delta \to 0$, the streamwise coordinate scale $Re^{-1/2}\delta^{-3/2}L^*$ and the time scale $Re^{-1/2}\delta^{-5/2}L^*U_\infty^{*-1}$ are introduced. Such scaling laws are combined with the additional assumption that the longitudinal velocity perturbation amplitude is of the order of δU_∞^*, and the evolution of disturbances is then controlled by a free interaction. This leads immediately to the governing equation

$$\frac{\partial A}{\partial t} + A\frac{\partial A}{\partial x} = -\frac{1}{\pi}\frac{\partial^2}{\partial x^2}\int_S\int \frac{\partial A(\eta,\xi)}{\partial \xi}\frac{d\eta d\xi}{\sqrt{(t-\eta)(x-\xi) - K_\infty(t-\eta)^2}}. \tag{1}$$

Here $A(t, x)$ can be regarded as a displacement thickness of the boundary layer. The integration domain S is given by the inequalities

$$S:\ \eta < t, \quad \xi < x - K_\infty(t-\eta).$$

If the transonic similarity parameter $K_\infty \to +\infty$, equation (1) transforms into the Burgers equation

$$\frac{\partial A}{\partial t} + A\frac{\partial A}{\partial x} = -\frac{1}{\sqrt{K_\infty}}\frac{\partial^2 A}{\partial x^2}.$$

In the opposite limit $K_\infty \to -\infty$, equation (1) reduces to the Benjamin-Ono equation

$$\frac{\partial A}{\partial t} + A\frac{\partial A}{\partial x} = \frac{1}{\pi\sqrt{-K_\infty}}\int_{-\infty}^{\infty}\frac{\partial^2 A(t,\xi)}{\partial \xi^2}\frac{d\xi}{\xi - x}.$$

As an example, the analytical solution of the non-linear integro-differential equation (1) is

$$A = c + \frac{k}{(c - K_\infty)^{1/2}}\left[\frac{1+\sigma^2}{1-\sigma^2} - 2\,\frac{1-\sigma^2}{1+\sigma^2 - 2\sigma\cos[k(x-ct)]}\right]. \tag{2}$$

The four independent parameters c, k, σ, K_∞ in equation (1) are subject to the constraints $0 < \sigma < 1$, $c > K_\infty$, $k > 0$. The periodic solution (2) can be represented as a superposition of repeated equidistant solitons. It is easy to check that the solitary wave solution

$$A = \frac{4c}{1 + c^2(c - K_\infty)(x - ct)^2}$$

satisfies equation (1).

The validity of the governing equation (1) corresponds to a four-deck asymptotic structure in the disturbed boundary layer. The nonlinearity is contained within a region near the wall. In the case $Re^{-1/9} \ll \delta \ll 1$, this region subdivides into an outer inviscid part and an inner viscous sublayer.

The displacement function A(x,t) gives the slip velocity at the bottom of the inviscid zone. Periodicity conditions of a regular solution in the above-mentioned viscous sublayer, driven by the edge velocity (2) then establish a relation between the four parameters:

$$k = c(c - K_\infty)^{1/2}(1 - 10\,\sigma^2 + \ldots). \tag{3}$$

The expression (3) can be interpreted as a generalization of the dispersion relation in linear stability theory to the finite-amplitude case. In the limit $\sigma \to 0$, the solution (2) describes an infinitesimal Tollmien-Schlichting wave and the dependence (2) approaches the neutral curve of transonic-flow stability.

THE INFLUENCE OF TEMPERATURE AND PRESSURE ON BOUNDARY LAYER STABILITY

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Classical linear stability theory is extended to include the effect of temperature and pressure dependent fluid properties. These effects are studied asymptotically. The basic approach starts from a Taylor series expansion of the properties with respect to temperature and pressure. Next, a regular perturbation is applied to the basic equations of stability with the constant property case representing leading order behavior governed by the Orr-Sommerfeld (OS) equation. In this asymptotic approach, all effects are well separated from each other, and only the Prandtl number remains as a parameter. In their general form the asymptotic solutions hold for all Newtonian fluids.

In the method of small disturbances, all quantities are decomposed in a mean value, $\bar{a}^*$, and a superimposed disturbance a'^*. Here a^* represents the velocity components u^* and v^* (two dimensional flow) and the pressure p^*. When variable properties are involved, it also represents these properties, i.e. density ρ^*, viscosity μ^*, thermal conductivity k^* and specific heat c_p^*, as well as the temperature T^*. Due to the temperature dependence of the properties, the modified OS equation must be supplemented by the thermal energy equation of the disturbance.

With a^* representing one of the physical properties, the Taylor series expansion reads

$$a = \frac{a^*}{a_R{}^*} = 1 + \epsilon K_{aT}\Theta + \tilde{\epsilon} K_{ap} p + O(\epsilon^2, \tilde{\epsilon}^2, \epsilon\tilde{\epsilon}),$$

with

$$\epsilon = \frac{\Delta T_R{}^*}{T_R{}^*}, \tilde{\epsilon} = \frac{\rho_R{}^* U_R{}^{*2}}{p_R{}^*}, K_{aT} = \left[\frac{\partial a^*}{\partial T^*}\frac{T^*}{a^*}\right]_R, K_{ap} = \left[\frac{\partial a^*}{\partial p^*}\frac{p^*}{a^*}\right]_R.$$

Here ϵ and $\tilde{\epsilon}$ are introduced as small (perturbation) parameters. The Taylor series are truncated after the linear terms. When the series are continued to higher orders, additional K_a – values appear which contain second and mixed derivatives for the next higher order. K_{aT} and K_{ap} are properties of the fluid. Owing to the decomposition $a = \bar{a} + \hat{a}\exp[i\alpha(x - \tilde{c}t)]$, the mean value and amplitude function are

$$\bar{a} = 1 + \epsilon K_{aT}\bar{\Theta} + \epsilon K_{ap}\bar{p} + O(\epsilon^2, \tilde{\epsilon}^2, \epsilon\tilde{\epsilon}), \quad \hat{a} = \epsilon K_{aT}\hat{\Theta} + \tilde{\epsilon} K_{ap}\hat{p} + O(\epsilon^2, \tilde{\epsilon}^2, \epsilon\tilde{\epsilon}).$$

This suggests the following expansion of all mean flow and disturbance quantities:

$$a = a_0 \quad + \epsilon(K_{\rho T}\, a_{1\rho} + K_{\mu T}\, a_{1\mu} + K_{kT}\, a_{1k} + K_{cT}\, a_{1c})$$

$$+\tilde{\epsilon}(K_{\rho p}\,\tilde{a}_{1\rho}+K_{\mu p}\,\tilde{a}_{1\mu}+K_{kp}\,\tilde{a}_{1k}+K_{cp}\,a+{}_{1c}) + O(\epsilon^2, \tilde{\epsilon}^2, \epsilon\tilde{\epsilon}),$$

where a represents: $\bar{u}$, $\hat{u}$, $\bar{v}$, $\hat{v}$, $\bar{p}$, $\hat{p}$, $\overline{\Theta}$, $\hat{\Theta}$, $\hat{\varphi}$, $\hat{c}$. Inserting these expansions into the basic stability equations and collecting terms with respect to $\epsilon K_{\rho T}$, $\epsilon K_{\mu T}$, etc., gives the asymptotic equations for the temperature and pressure influence.

This method is applicable to all flow situations where temperature and pressure variations of the physical properties involved are important. Various examples will be given.

NONLINEAR ASYMPTOTIC SOLUTIONS OF FLOW PAST THIN BODIES ON ENTRY INTO A COMPRESSIBLE FLUID

by

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Linear solutions of problems on the flow past thin bodies entering a fluid have a major drawback since they diverge near the vertex of the body; this makes it impossible to determine the flow characteristics in the neighborhood of the vertex and, most importantly, the magnitude of the maximum pressure at the nose of the body. Here, by the method of matched asymptotic expansions, we find a composite solution of the problems of the flow past, and the entry of, thin bodies into a compressible fluid (for incompressible fluids, see Gonor 1986, 1991). The new solution is based on considering nonlinear terms in the Cauchy-Lagrange (Bernoulli) integral and is uniformly valid in a neighborhood of the nose of the body.

<u>Formulation of the problem.</u> A thin body (plane or axisymmetric) with half-angle $\epsilon \ll 1$ at the vertex enters a fluid half-space as shown in Figure 1. In a fixed coordinate system, the initial boundary-value problem reduces to the solution for the potential $\varphi(x, y, t)$ of the wave equation with the following boundary and initial conditions: on the wetted surface of the body $\partial_n \varphi = v_o(t) \sin \alpha$, where $v_o(t)$ is the speed of the body; on the free surface $x = f(y, t)$: $\varphi_t + \nabla^2 \varphi/2 = 0$. The equation of the free surface $x = f(y, t)$ is determined by the kinematic condition $f_t = \varphi_x - f_y \varphi_y$ at $x = f(y, t)$. The initial conditions for the potential are $\varphi(x, y, 0) = \varphi_t(x, y, 0) = 0$. The pressure is connected with the potential by the Cauchy-Lagrange integral.

<u>The outer solution for a thin cone.</u>

We take the half-angle ϵ at the vertex of the body as a small parameter in terms of which the desired solution will be found by a series expansion.. We decompose the zone of the perturbed flow into three regions as shown in Figure 1, the last two of which are in a neighborhood of the points A and O respectively. In each of the regions, we choose a scale for the independent and dependent variables. For region (1), we have $x \sim y \sim 1$, $\varphi \sim \epsilon^2$ (or ϵ for plane bodies). We seek the potential $\varphi(x, y, t)$ in the form of a series,

$$\varphi = \sum_{n=0}^{\infty} \epsilon^{n+2} \varphi_n(x, y, t).$$

The leading term of this expansion is determined by solving a linear problem. We take the flow potential of the linear problem as the outer solution. For the cone with half-angle ϵ, we have

$$\varphi_0 = 0.5\cdot\epsilon^2 v_o\cdot\left\{(v_o t\text{-}x)\ \ln\left|\frac{(x-\xi_2)+[(x-\xi_2)^2+y^2]^{1/2}}{x+(x^2+y^2)^{1/2}}\right|\right.$$

$$+\ (v_o t+x)\cdot\ln\ \left|\frac{(x-\xi_1)+[(x-\xi_1)^2+y^2]^{1/2}}{x+(x^2+y^2)^{1/2}}\right| + M(\xi_1+\xi_2)+[(x-\xi_2)^2+y^2]^{1/2}$$

$$\left.-\ [(x-\xi_1^2)+y^2]^{1/2}\right\},$$

$$\xi_2(M^2-1) = M^2\,x - v_o\,t + M\cdot\ [(v_o t - x)^2+(1-M^2)y^2]^{1/2}\ ,$$

$$\xi_1(M^2-1) = M^2\,x + v_o\,t + M\cdot\ [(v_o t + x)^2+(1-M^2)y^2]^{1/2}.$$

Here v_0 is a constant.

The inner solution in a neighborhood of the vertex of the body.

The region of inhomogeneity of the outer solution φ_o has the characteristic dimension $\sim e^{-1/\epsilon}$ and we therefore introduce new scales according to the formulae: $x-x_o(t)=x_1 e^{-1/\epsilon}$, $y=y_1 e^{-1/\epsilon}/\beta$, $\varphi=\varphi_i e^{-1/\epsilon}$, where the arbitrary motion of the vertex of the body is given by a function $x_o(t)$, $\beta=(1-M^2)^{1/2}$. Passing to interior variables, we seek the potential of the inner solution of the Laplace equation. We write the potential of the absolute motion of the fluid as a sum of the potentials of the transport and the relative motion $\varphi_i = v_o x_1 + \Phi(x_1, y_1, t)$. It is convenient to seek the potential Φ in spherical coordinates. A solution of the Laplace equation in spherical coordinates (Θ, R, Ψ) can be written in the following form $\phi(\Theta, R, t) = U(t)R^n P_n(\cos\Theta)$. Here $U(t)$ is an arbitrary function of time and $P_n(\cos\Theta)$ is the Legendre function of arbitrary (fractional) power, defined from the boundary condition on the surface of the cone. As a result, we have a Gilbert problem with the inclined derivative, which reduces to

$$nP_n(\cos\Theta)(\epsilon_1\cos\Theta\ +\ \sin\Theta)+P_n'(\cos\Theta)\cdot C\cos\Theta-\epsilon_1\sin\Theta) = 0$$

$$\Theta = \pi - \Theta_o,$$

where $\Theta_o = \epsilon\cdot\beta$ is a small quantity. To obtain the inner solution of the problem, an explicit representation of the Legendre function is required. The known integral expressions and hypergeometric expansion for the Legendre functions do not enable us to realize this possibility.

A new asymptotic representation of Legendre functions.

We represent the power n as a sum of an integer and a fraction $(n = n_o + m)$. We assume the quantity m to be a small parameter (clearly $|m| \le 0.5$) and seek a solution of the Legendre equation $z = P_n(\cos\Theta)$ in the form of a power series

$$z = \sum_{k=0}^{\infty} m^k z_k.$$

The terms of the series are found successively by solving the sequence of equations

$$(1-M^2)\, z_k'' - 2\mu z_k' + n_o(n_o+1)\, z_k = -z_{k-2}U_-\,(k-2) - (2n_o+1)\, U_-\,(k-1),$$

where $U_-(x)$ is the asymmetric identity function and $\mu = \cos\Theta$. The leading term of the expansion is the Legendre polynomial $z_o = P_{n_o}(\mu)$. Here we are interested in a Legendre function with fractional power close to one. In this case $n_o = 1$, $z_o = P_1\ (\mu) = \mu$. As a result, $P_{1+m}(\cos\Theta) = \cos\Theta + m[\cos\Theta \cdot \ln(1+\cos\Theta) + (1-\ln 2)\cos\Theta - 1] + \ldots$.

Construction and matching of the inner and outer solutions.

We differentiate the expression $P_{1+m}(\cos\Theta)$ and use the above boundary condition. Noting that $n = 1+m$ and $\Theta_o = \epsilon \cdot \beta$, we obtain $n = 1 + 0.5\epsilon^2(1+\epsilon^2 \ln|0.5\,\epsilon\beta|)$. Substituting the expression $P_n(\cos\Theta)$ into the formula for potential, we find the potential φ_i determining explicitly the inner solution

$$\varphi_i = v_o(t)x_1 + U(t)\cdot(x_1^2+y_1^2)^{(2+\epsilon^2)/4}\Big\{\cos\Theta + \frac{\epsilon^2}{2}[\cos\Theta \ln|1+\cos\Theta|$$

$$+ (1+\ln 2)\cos\Theta - 1\Big\} + C(t).$$

The arbitrary functions $U(t)$ and $C(t)$ are found from the conditions of matching with the outer solution. In final form the composite solution φ_c is found from the formula

$$\varphi_c = v_o(x-x_o)[1 + 0.5\epsilon^2(2(1-M) - \ln|4x_o(1-M)/(1+M)|)]$$

$$+ 0.5\,\epsilon^2\, v_o\Big\{(x+x_o)\ln\big|x+x_o+[(x+x_o)^2+\beta^2 y^2]^{1/2}\big|$$

$$-2x_o \ln\big|\beta(x+(x^2+y^2)^{1/2})\big| - [(x+x_o)^2+\beta^2 y^2]^{1/2}$$

$$+ x(\ln|(1-M)/(1+M)| + 2M)\Big\} + U(t)\, e^{\epsilon/2}\,[(x-x_o)^2+\beta^2 y^2]^{\epsilon^2/4}$$

$$\times\Big\{x - x_o + 0.5\epsilon^2\,[(x-x_o)\ln|(x-x_o+[(x-x_o)^2+\beta^2 y^2]^{1/2})/$$

$$[(x-x_o)^2+\beta^2 y^2]^{1/2}| + (1-\ln 2)(x-x_o) - [(x-x_o)^2+\beta^2 y^2]^{1/2}]\Big\}.$$

The potential φ_c determines a solution suitably uniform both near and away from the vertex of the cone. By means of the Cauchy-Legendre integral, it is not hard to determine, in particular, the pressure at the vertex of the cone,

$$C_p = (p_1 - p_o)/0.5\,\rho_o v_o^2 = 1 + 2\epsilon^2\,[1 - M + \ln|\beta(1+M)/2|\,].$$

Here the second term in square brackets characterizes the pressure increase as compared with the stagnation pressure in the steady-state case. For a wedge this increment depends linearly on the angle. The problem of subsonic flow past thin bodies is solved analogously.

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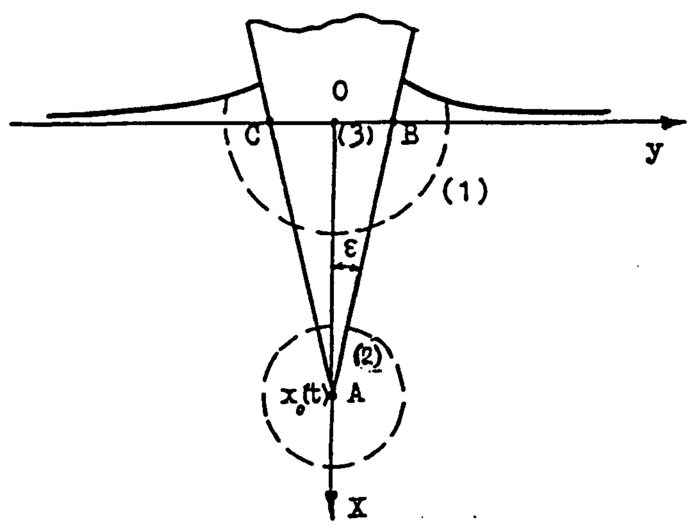

Фиг.I

Figure 1

CURVATURE EFFECTS AND STRONG VISCOUS-INVISCID INTERACTIONS

by

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Asymptotic theory, in particular the triple-deck theory as developed by Stewartson and Messiter around 1970, has laid the foundation for one of the more successful viscous-inviscid interaction methods: the quasi-simultaneous method. Triple-deck theory describes the structure of the flow field near singular points like the trailing edge or a point of separation. The message of this theory is three-fold.

- Near a single point, a smaller length scale in streamwise direction exists. This scale has to be reflected in a finer distribution of grid points in a numerical solution method.
- The importance of the various terms in the equations of motion is indicated, with result that in first approximation other classical shear-layer equations are sufficient to describe the flow in the vicinity of the singular point.
- The interaction between the shear layer and the outer inviscid flow can be described by thin airfoil theory. Furthermore there is no hierarchy between the viscous shear layer and the outer inviscid flow: this is called strong interaction. This has to be reflected in the interaction process for solving the flow equations.

The quasi-simultaneous method has been designed based on these three messages. We will describe it here in terms of the viscous pressure distribution p_e and the displacement thickness δ^*. Let the external flow be described by $p_e = E[\delta^*]$ (where E denotes, for example, a transonic full-potential equation), and let the shear layer be described by $p_e = B[\delta^*]$ (where B represents the shear-layer equations). Then the quasi-simultaneous iterations are given by

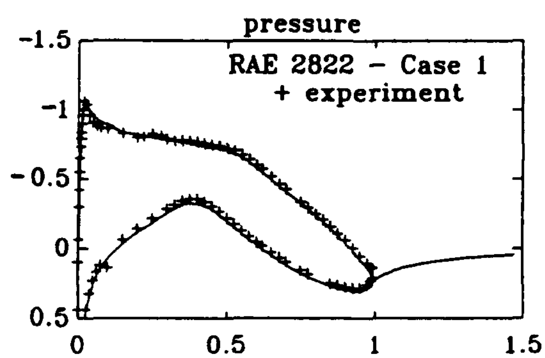
$$\begin{cases} p_e^{(n+1)} - I[\delta^{*(n+1)}] = E[\delta^{*(n)}] - I[\delta^{*(n)}], \\ p_e^{(n+1)} - B[\delta^{*(n+1)}] = 0. \end{cases} \tag{1}$$

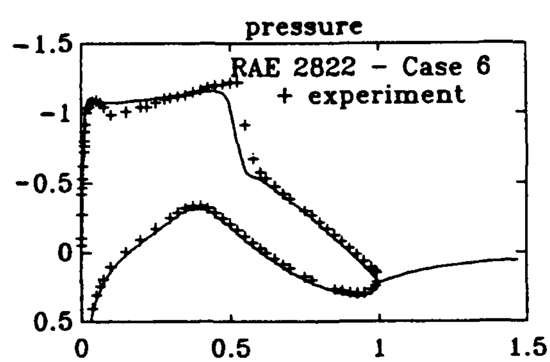
The lack of hierarchy strongly suggests a simultaneous iterative treatment of the viscous and inviscid flow equations. For convenience reasons only, the relevant part of the interaction is treated simultaneously. This part is described by the interaction law $p_e = I[\delta^*]$, where I is based on thin-airfoil theory.

In the current presentation, we will extend the modeling of the shear-layer equations. The streamlines immediately behind the trailing edge are highly curved, especially for rear-loaded airfoils. In such a situation, the assumption of constant pressure across the shear layer is no longer acceptable. In asymptotic terms it means that higher-order effects, in regions even smaller than the triple-deck, are becoming relevant. We will demonstrate that this curvature effect again is a strong interaction effect. This explains numerical difficulties encountered when this effect is not treated in a simultaneous way; extensive smoothing is then required in order to obtain convergence of the viscous-inviscid iterations.

In a simultaneous treatment of the curvature effect, no iterative difficulties occur. We will discuss this at length using a model problem. Thereafter we will demonstrate this for realistic calculations: for transonic flow past an RAE 2822 airfoil (flow cases 12, 6 and 9), a handful of quasi-simultaneous iterations (1) suffices.

Convergence of viscous-inviscid iterations.





2

THE SYMMETRY OF ISENTROPIC IDEAL GAS FLOWS

by

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The classical model M_3 of unsteady isentropic ideal (polytropic) gas flows is considered. The system of differential equations descriptive of this model is subjected to group theory analysis in order to show its possibilities for finding classes of exact solutions.

The initial symmetry of the model M_3 is to admit the full group G_8 generated by translations along coordinate axes, rotations and two extensions added with two discrete reflection symmetries. The goal is achieved by constructing a tree (the optimal system with inclusions) of all non-similar subgroups of the group G_8. This makes it possible to list all non-equivalent exact submodels, namely, classes of invariant solutions described by differential equations with a smaller number of independent variables. This number is called the submodel's rank and may be equal to two, one, or zero. Each of the second rank submodels is subjected to the individual group analysis to determine if additional invariant submodels of the rank one or zero arise.

The classes of the partially invariant irreducible solutions are listed and are also described by the simplified differential equations. Questions concerning the differentially-invariant solutions are discussed. The searching procedure and the classification call for the independent group analysis are described. The peculiarities of the appropriate gas flows are pointed out for some new exact submodels of the model M_3.

NUMERICAL SOLUTION FOR A CRISS-CROSS INTERACTION PROBLEM

by

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The asymptotic theory of laminar-boundary layer separation, proposed by Neiland (1969), Stewartson and Williams (1969), and Sychev (1972), has been used to solve a large number of problems. Two-dimensional (2D) flows with separation and interaction between the boundary layer and external flows were mainly investigated, but only a few publications were devoted to three-dimensional (3D) theory. A 3D interaction phenomenon was first considered in an application to a hypersonic boundary layer when the induced pressure gradient appears to be essential over the entire body surface (Kozlova and Michailov, 1970; Ruban and Sychev, 1973). In subsonic and supersonic flows, the interaction region is local and occupies a small neighborhood of the separation point, with longitudinal extent of the order of $Re^{-3/8}$ where Re is Reynolds number. To investigate 3D effects on the flow behavior in the interaction region, Smith, Sykes and Brighton (1977) considered a Blasius 2D boundary layer encountering a 3D roughness on the flat-plate surface. They constructed an interaction theory for the case when the width of the roughness is of the same order of magnitude as its extent, $O(Re^{-3/8})$. Subsequent investigations revealed that different regimes of interaction are possible, depending on the longitudinal and lateral scales of the roughness.

The present paper is devoted to a numerical analysis of the so-called 'criss-cross' interaction regime proposed by Rozhko and Ruban (1987). In contrast to their investigation, the nonlinear interaction problem with 3D boundary-layer separation is considered. To carry out the corresponding calculations, a spectral method (Duck and Burggraf, 1986) based on the Fast Fourier Transform (FFT) is used.

Let us consider a 2D boundary layer on a curved surface encountering a 3D roughness (see Figure 1). In the neighborhood of the roughness, the triple-deck interaction region is forming. In order to take into account the pressure variation across the boundary layer,* we assume that the extent of the surface roughness is $\Delta x = O(Re^{-3/14})$ and its lateral scale is $\Delta z = O(Re^{-3/7})$. Then the flow inside the lower deck, the viscous sublayer, may be described by the boundary-layer equations without a longitudinal pressure gradient,

$$
\begin{aligned}
&U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} + W\frac{\partial U}{\partial Z} = \frac{\partial^2 U}{\partial Y^2},\\
&U\frac{\partial W}{\partial X} + V\frac{\partial W}{\partial Y} + W\frac{\partial W}{\partial Z} = -\frac{\partial P}{\partial Z} + \frac{\partial^2 W}{\partial Y^2},\\
&\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0.
\end{aligned}
\tag{1}
$$

* More precisely, across the middle deck of the interaction region.

If the height of the roughness is of the order of $\Delta y = O(Re^{-4/7})$, the equations remain nonlinear and the boundary conditions may be expressed in the form

$$U = V = W = 0 \quad \text{at} \quad Y = F(X,Z),$$

$$U = Y + A(X,Z) + \ldots, \quad W = D(X,Z)\cdot Y^{-1} \quad \text{as} \quad Y \to \infty,$$

$$U \to Y, \; W \to 0 \quad \text{as} \quad X \to \infty,$$

where the function F(X,Z) is introduced to describe the shape of the roughness.

The pressure gradient is not known in advance but may be obtained via the interaction law

$$\frac{\partial P}{\partial Z} = \mathrm{sign}(\kappa_o)\frac{\partial A}{\partial Z} - \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\partial^2 A(X,\xi)}{\partial x^2}\frac{d\xi}{\xi - Z}. \tag{2}$$

The first term in (2) is responsible for the pressure variation across the boundary layer and the second describes the pressure induced in the external inviscid flow due to the displacement thickness of the boundary layer.

The linearized problem for small roughness heights was considered in Rozhko, Ruban and Timoshin (1988). To simplify the problem, Rozhko, Ruban and Timoshin assumed also that the extent of the roughness is much greater than $Re^{-3/14}$. In that case, only the first term is (2) and should be used to determine the pressure distribution over the interaction region. The present investigation is aimed at providing general a solution for (1), (2).

We introduce the Fourier Transform of the unknown functions according to

$$U^{**}(k, \ell, Y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dX \int_{-\infty}^{\infty} U(X, Y, Z)\exp(-ikX - i\ell Z)dZ.$$

Then the boundary-layer equations may be written in the form

$$\frac{d^3 f}{dY^3} - ikY\frac{df}{dY} = \frac{d}{dY}\,(kR_1^{**} + \ell R_2^{**}). \tag{3}$$

The boundary conditions for (3) are

$$f = 0,\; \frac{d^2 f}{dY^2} = i\ell^2 P^{**} \quad \text{at} \quad Y = 0, \qquad f|_{Y=\infty} = k(A^{**} + F^{**}).$$

Here

$$f = kU^{**} + \ell W^{**}, \quad R_1 = U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}, \quad R_2 = U\frac{\partial W}{\partial X} + V\frac{\partial W}{\partial Y} + W\frac{\partial W}{\partial Z}.$$

The interaction law (2) takes the form

$$P^{**} = [\mathrm{sign}(\kappa_o) + k^2/|\ell|]A^{**}.$$

The solution is sought by means of the method of successive approximations with the right-hand side in (3) taken from the previous iteration. The Thomas algorithm in the Y direction combined with the FFT procedure is used to obtain the solution. Calculations were carried out for a hollow of the form

$$F(X,Z) = h_o \exp(-\alpha X^2 - \beta Z^2).$$

Figure 2 illustrates the skin friction lines for the particular hollow with $h_o = -5.3$, $\alpha = \beta = 1/2$.

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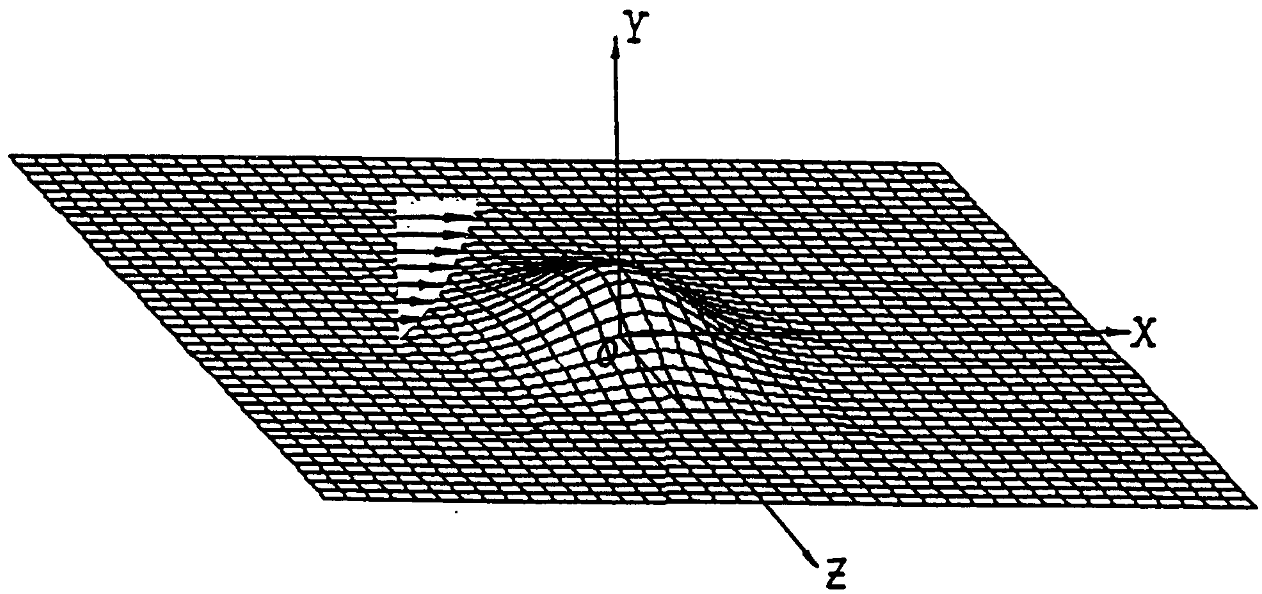

Figure 1

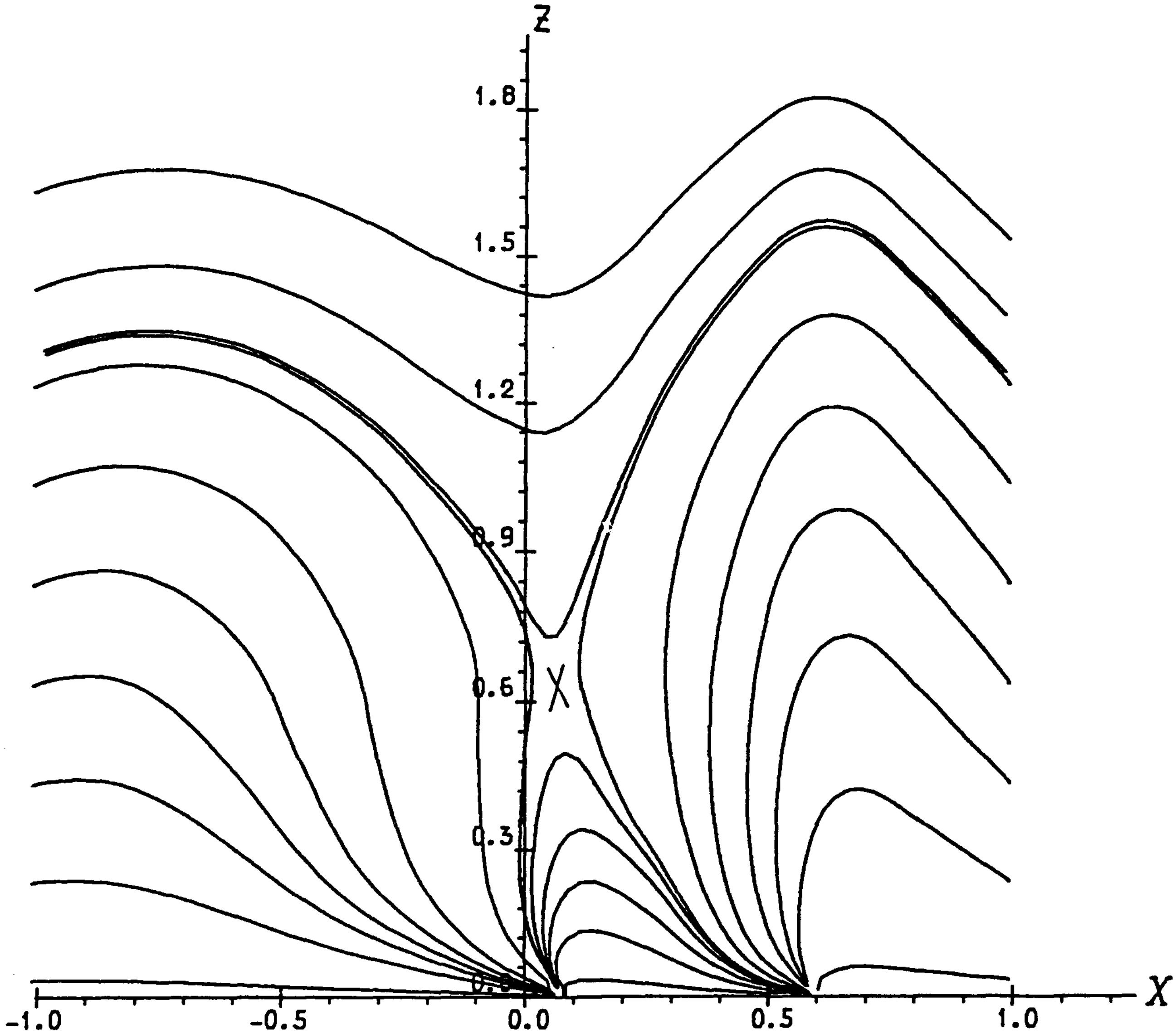

Figure 2

THE BEHAVIOR OF SELF-SIMILAR SOLUTIONS OF BOUNDARY LAYER WITH ZERO PRESSURE GRADIENT

by

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The present paper deals with the self-similar solutions of the boundary layer equations with zero pressure gradient. The similarity solutions describe the flow in a mixing layer (as well as the Blasius boundary layer), since away from the separation point the mixing layer forms mainly under the effect of the boundary conditions and is weakly dependent on initial conditions. The similarity function $\Phi(\zeta;m)$(where ζ is the self-similar variable and m is a parameter) satisfies a well-known nonlinear differential equation of third order

$$\frac{(m-1)}{m}\left(\frac{d\Phi}{d\zeta}\right)^2-\Phi\frac{d^2\Phi}{d\zeta^2}=\frac{d^3\Phi}{d\zeta^3}, \tag{1}$$

$$\zeta=[m/(m+1)]^{1/2}\, y/x^{1/(m+1)},\ m>0.$$

The equation (1) coincides with the Blasius equation at $m=1$.

Although three-point boundary-value problems in the classic problem of mixing layers were posed more than half a century ago, there has been no satisfactory investigation of the behavior of their solutions until now. The interest in self-similar solutions has re-arisen after fundamental results obtained from free-interaction theory. This theory provides a way of describing the structure of the flow in the vicinity of the separation point. As a result, a number of new nonclassical problems have emerged that are realized for values of $m \epsilon$ in $(1, 2)$, $m=\infty$ and have to be studied to provide a strict foundation for the obtained result. The case $m=\infty$ arises from unsteady separation theory. All the boundary problems that describe the weak and mixing layer flows can be associated with three different groups. It is shown that (1) is invariant in relation to a shift transformation and a similarity transformation. It permits reduction of (1) to a first-order equation solved with respect to the derivative. The right-hand part of this equation equals the ratio of two second-order polynomials,

$$\frac{d\Psi}{dF}=-\frac{\Psi^2+7F\Psi+6F^2+\Psi+[(m+1)/m]F}{F\Psi}, \tag{2}$$

$$\frac{d\Phi}{d\zeta}=\Phi^2F,\quad \Phi\frac{dF}{d\Phi}=\Psi.$$

Equation (2) has three singular points $A(0, 0)$, $B(0,-1)$, $C\left(\frac{m+1}{6m},0\right)$ in a finite part of the plane (F, Ψ) and three infinite singular points $E-E'$; $G-G'$; $Q-Q'$. On the projective single circle, these points are separated, and each of them has coordinates $E(1/\sqrt{5},\ -2/\sqrt{5})$, $G(2/\sqrt{13},\ -3/\sqrt{13})$, $Q(0,-1)$, $E'(-1/\sqrt{5},\ 2/\sqrt{5})$, $G'(-2/\sqrt{13},\ 3/\sqrt{13})$, $Q'(0,1)$. Hitting the points A, B, E-E' of the integral curves gives the possibility of satisfying the boundary conditions in the boundary-value problems considered. Thus the problem concerning behavior of the thrice-continuously-differentiable solutions $\Phi(\zeta;m)$ of (1) changes to the investigation of the behavior of the integral curve of the first order equation (2) and its singular points. For studying (2), the methods of qualitative analysis advanced by Poincare, Bendixson, Frommer and Haimmov have been applied. This has allowed research of the behavior of all the integral curves $\Phi(\zeta;m)$ and, in particular, consideration of the question regarding the existence and uniqueness of the solution of the two- and three-point problems arising in the wake and mixing layer theory. It has also been demonstrated that (1) has no exponentially growing solutions.

We do not consider the values of the parameter $m < 0$. The behavior of the integral curves is distinguished essentially for $m > 1$ and $0 < m \leq 1$ (see Figures 1, 2). For $0 < m \leq 1$ their picture is formed so that, in the boundary problems considered, there are no solutions with inverse velocities. For $m = 2$ and the condition $\Phi'(0) > 0$, it is established that we have the unique solution describing the Goldstein wake. It is proven that if this condition is lacking, there is yet another sole solution with inverse velocities satisfying all the prescribed boundary conditions. A similar situation occurs in the problem of the mixing of two streams leaving the trailing edge of the plate with different velocities.

Note that the Goldstein solution is depicted by the curve Ψ_E in the plane (F, Ψ) (see Figure 2). The Chapman solution is depicted by the curves Ψ_1 and Ψ_2 (see Figure 1).

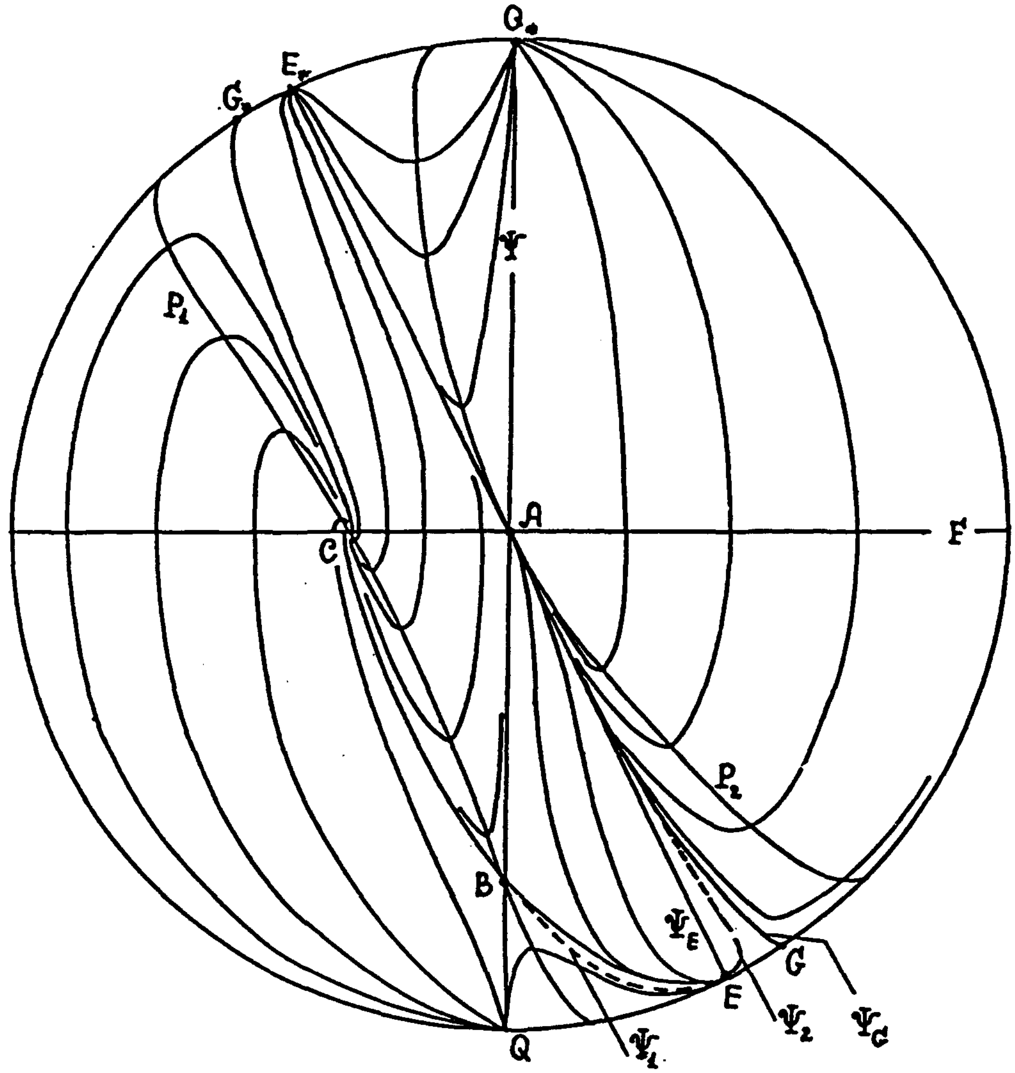

Figure 1 (m = 1)

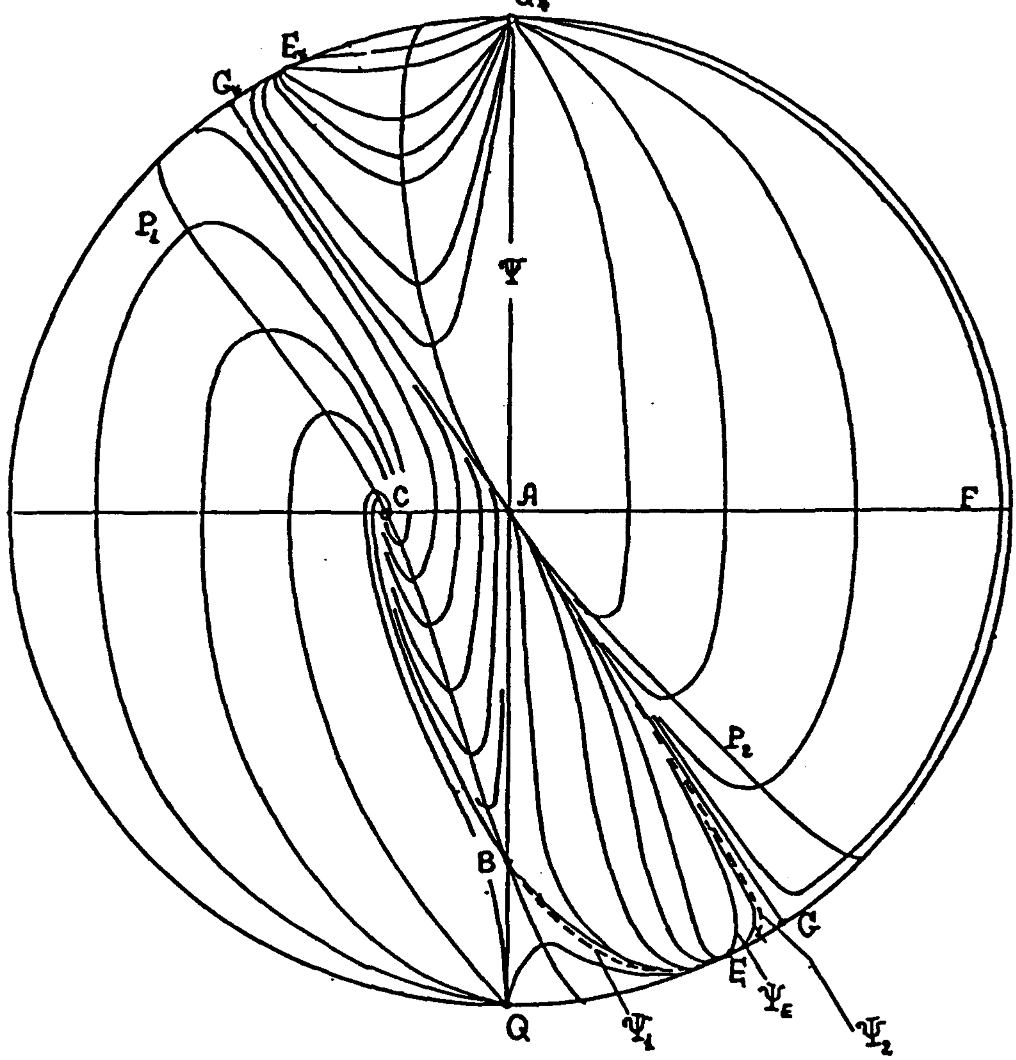

Figure 2 (m = 2)

PARAMETRIC REPRESENTATION OF EXACT SOLUTIONS OF THE TRANSONIC EQUATIONS

by

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Some possibilities for parametric representation of exact solutions of the transonic equations of a perfect gas, in the plane and axisymmetric cases, are investigated. Both the functions and the independent variables are considered as rational functions of a first parameter, with coefficients dependent on a second parameter. The system of ordinary differential equations for the coefficients is deduced from the transonic equations and is superdeterminate in general. The method is used for finding a number of singular solutions in the axisymmetric case. The main points of the method are the following: (i) the consideration of the solution in the plane case using the hodograph plane; (ii) the representation of this solution in parametric form; and (iii) the construction of an axisymmetric analogue. Finally, the analogues are found for the flows over the Guderley profile (the flow over a finite body with a sonic flow at infinity) and the Tomotika-Tamada profile (with local supersonic zones).

The connection of this method with the technique used in the analysis of groups of the differential equation is also discussed.

1. Transonic flows of an ideal gas in the plane ($\omega=0$) and axisymmetric ($\omega=1$) cases are described by the Kármán equations

$$u \cdot u_x = v_y + \omega \frac{v}{y}, \quad u_y = v_x . \tag{1.1}$$

Let us consider the class of particular solutions of (1.1) in the form

$$y=t, \; u=u_0+u_2(s)t^2, \; v=v_1(s)t+v_3(s)t^3, \; x=x_0(s)+x_2(s)t^2 , \tag{1.2}$$

where s and t are parameters. For the coefficients u_0, u_2, v_1, v_3, x_0, x_2, the system of ordinary differential equations (ODEs) turns out to be, from (1.2),

$$\begin{aligned} &x_2' = u_2 - 4x_2^2, \; u_2' = (1+\omega)v_3 - 4x_2u_2, \; v_3' = 2(u_2^2 - (1+\omega)x_2v_3) , \\ &x_0' = u_0, \; u_0' = (1+\omega)v_1, \; v_1' = 2(u_0u_2 - (1+\omega)x_2v_1) . \end{aligned} \tag{1.3}$$

Three equations for x_2, u_2, v_3 form an independent nonlinear subsystem; this is the nonlinear kernel of (1.3). It has been studied in the theory of similar transonic flows of the form

$$u = y^{2(n-2)}\, U(\xi), \; v = y^{3(n-1)}\, V(\xi), \; \xi = xy^{-n} \tag{1.4}$$

with $n=2$. The other solutions are expressed through $u_2(x_2)$, $v_3(x_2)$ in the form of integrals.

The general solution of (1.3) contains five constants. Choosing them, one can obtain a number of interesting solutions which are relevant to aerodynamics, as follows.

a) Meyer similar flow in a Laval nozzle:

$$u = C_1 x + \frac{C_1^2 y^2}{2(1+\omega)}, \quad v = \frac{C_1^2}{1+\omega} xy + \frac{C_1^3}{2(3+5\omega)} y^3. \tag{1.5}$$

b) Taylor nozzle flow with local supersonic zones ($\omega = 0$):

$$u = \frac{C_1(1 - C_2 s^2)}{2s^2} + \frac{C_1^2}{4} y^2, \quad v = y\left[\frac{C_1(2 + C_2 s^3)}{4s^2} - \frac{C_1^3}{12} y^2\right],$$
$$x = \frac{1 + 2C_2 s^3}{2s^2} - \frac{C_1}{4} y^2. \tag{1.6}$$

It is easy to obtain the axisymmetric analogue of (1.6).

c) Similar Frankl flow ($n = 4/5$) and the Guderley-Yoshihara flow ($n = 4/7$) (which apply far from the profile shape and the body of revolution for $M_\infty = 1$):

$$\omega = 0: u = -2C_1 s^{-1} + s^{-6} y^2, \quad v = y\left[2C_1 s^{-4} - \frac{2}{3} s^{-9} y^2\right],$$
$$x = -C_1 s^2 + s^{-3} y^2. \tag{1.7}$$

$$\omega = 1: u = -4C_1 s^{-3} + \frac{2}{3} s^{-10} y^2, \quad v = y\left[4C_1 s^{-8} - \frac{4}{9} s^{-15} y^2\right],$$
$$x = -3C_1 s^2 + s^{-5} y^2. \tag{1.8}$$

d) Vaglio-Laurin flow and its axisymmetric analog (flow over a corner point):

$$\omega = 0: u = -5C_1 s^2 + s^{-6} y^2, \quad v = y\left[-10C_1 s^{-1} - \frac{2}{3} s^{-9} y^2\right],$$
$$x = -C_1 s^5 + s^{-3} y^2 \tag{1.9}$$

$$\omega = 1: u = -42C_1 s^2 + \frac{2}{3} s^{-10} y^2, \quad v = y\left[-28C_1 s^{-3} - \frac{4}{9} s^{-15} y^2\right],$$

$$x = -9C_1 s^7 + s^{-5} y^2. \tag{1.10}$$

2. The possibility of linearization of (1.1) exist for $\omega = 0$, by the hodograph method. The solution of (1.3) is represented as

$$x = x(u,\ v,\ C_\kappa), \quad y = y(u,\ v,\ C_\kappa), \quad \kappa = 1,\dots,5 \tag{2.1}$$

The class of solutions of (1.1) associated with (2.1) may be considered. Let us introduce the operators of differentiation by,

$$\frac{\partial}{\partial v} = \frac{u_s \partial/\partial t - u_t \partial/_s \partial s}{u_s v_t - u_t v_s},$$
$$\frac{\partial}{\partial C} = \frac{\partial}{\partial C} + \frac{(v_C u_t - u_C v_t)\partial/\partial s + (u_C v_s - u_s v_C)\partial/\partial t}{u_s v_t - u_t v_s}, \tag{2.2}$$

where C is one of $C_\kappa(\kappa = 1,\dots,5)$. If (x,y) is a solution of the hodograph equations, then $(\partial x/\partial v,\ \partial y/\partial v)$, $(\partial x/\partial C,\ \partial y/\partial C)$ are solutions also. The parametric form

$$x = x(s,\ t,\ C_\kappa), \quad y = y(s,\ t,\ C_\kappa), \quad u = u(s,\ t,\ C_\kappa), \quad v = v(s,\ t,\ C_\kappa) \tag{2.3}$$

is assumed in the calculation of the right-hand sides of (2.2). The corresponding integration should be considered together with the differentiation in (2.2).

3. To obtain the flow over the Guderly profile, the solution (1.5) with $\omega = 0$ in parametric form should be written

$$u = Bs + \frac{B^2}{2}t^2, \quad v = B^2 st + \frac{B^3}{6}t^3, \ x = s, \quad y = t\,. \tag{3.1}$$

Using (2.2), it can be shown that

$$x_2 \equiv \partial^2 x/\partial v^2 = -\frac{B^6(s + Bt^2)}{K^3}, \quad y_2 \equiv \partial^2 y/\partial v^2 = \frac{2B^6 t}{K^3}, \quad K = B^3 s - \frac{B^4}{2}t^2. \tag{3.2}$$

The sum total $(x_2,\ y_2)$ from (3.2) together with u, v from (3.1) determines the Frankl solution (1.7). In fact, introducing the new parameters $z = k^{-1}$, $t_1 = 2B^6 z^3 t$ instead of (s, t), one can derive this solution in the same form ($z = s$, $t_1 = t$ are assigned again).

The flow over the Guderley profile is described with the help of a linear combination (x, y) from (3.1) and $(x_2,\ y_2)$ from (3.2) for identical (u, v),

$$u = \frac{1}{s} + \frac{C_1^2}{(2s^3 + C_2)^0}y^2, \quad v = \frac{y}{(2s^3 + C_2)}\left[\frac{C_1}{s} + \frac{2}{3}\frac{C_1^3}{(2s^3 + C_2)^2}y^2\right],$$
$$x = \frac{C_1(C_2 - 4s^3)}{2(2s^3 + C_2)^2}y^2 + \frac{C_2 - s^3}{C_1 s}\,. \tag{3.3}$$

Thus it has been shown that the flow over the Guderley profile is described by a solution of the form (1.2). It is easy to construct the axisymmetric analogue by solving (1.3) for $\omega = 1$.

4. Tomotika and Tamada demonstrated the crossover from the solution (1.6) to the singular one (like a transonic dipole). It describes the far flow over a profile with supercritical speed at infinity. It can be written in parametric form,

$$u = u_0 + u_2 t^2, \quad v = v_1 t + v_3 t^3, \quad x = \frac{x_0 + x_2 t^2}{z_0 + t^2}, \quad y = \frac{t}{z_0 + t^2}, \tag{4.1}$$

with

$$u_0 = -\frac{27C_1^3C_2s^3 - 1}{18C_1s^2}, \quad u_2 = s^2, \quad v_1 = \frac{27C_1^3C_2s^3 + 2}{18C_1s}, \quad v_3 = -\frac{2s^3}{3},$$
$$x_0 = -\frac{27C_1^3C_2s^3 + 2}{36C_1s^3}, \quad x_2 = s, \quad z_0 = -\frac{729C_1^6C_2^2s^6 + 108C_1^3C_2s^3 + 4}{216C_1s^4}. \tag{4.2}$$

Substituting (4.1) in (1.1), one can obtain a system of ODEs for the seven coefficients u_0, u_2, v_1, v_3, x_0, x_2, z_0 which are homogeneous and linear according to the derivatives of the coefficients. The condition of compatibility of this system is its determinant is zero.

$$\Delta(u_0, u_2, v_1, v_3, x_0, x_2, z_0) = 0\,. \tag{4.3}$$

It is easy to show if the initial conditions of the Cauchy problem for the system of ODEs are satisfied by (4.3), then they are also at every step of the integration (the property of involution). One of the seven equations may be discarded. So the solution (4.2) can be generalized for $\omega = 0$ (introducing new constants) and one can construct the axisymmetric analogue.

The Tomotika-Tamada profile is described by a linear combination of (4.2) and (1.6) in the hodograph plane. It gives the parametric form

$$u = u_0 + u_2 t^2, \; v = v_1 t + v_3 t^3, \; x = \frac{x_0 + x_2 t^2 + x_4 t^4}{z_0 + t^2}, \; y = \frac{t + y_3 t^3}{z_0 + t^2}. \tag{4.4}$$

A substitution of (4.4) into (1.1) gives overdeterminate systems of ODEs. One may study them by analogy with the ones mentioned above.

THREE-DIMENSIONAL LAMINAR BOUNDARY-LAYER ON A FINITE-DELTA WING IN VISCOUS INTERACTION WITH HYPERSONIC FLOW

by

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In hypersonic flow over thin delta wings, boundary-layer flow pattern depends essentially on the magnitude of the hypersonic interaction parameter $\chi = M_\infty^2 Re_o^{-1/2}$, where M_∞ is the incoming flow Mach number, $Re_o = \rho_\infty U_\infty L/\mu_o$ is the Reynolds number defined in terms of the free stream gas density and velocity, the wing length and the viscosity coefficient at the stagnation temperature. For $\chi \geq O(1)$, moderate or strong interactions are realized. In this case a pressure gradient, induced in the external inviscid flow by the boundary-layer displacement thickness, influences the boundary layer to leading order, and upstream disturbances can take place over the whole body length (Neiland, 1970). The problem of flow over a semi-infinite delta wing in a strong viscous interaction ($\chi \gg 1$) admits reduction of the boundary-value problem to a self-similar one, for which methods developed for the two-dimensional problem (Kozlova and Mikhailov, 1970; Dudin, 1978) are applicable. However, when considering moderate interactions for flow over a delta wing ($\chi = O(1)$), the system of equations for the boundary layer remains three-dimensional and the method presented, for example, in Dudin (1983) must be used to obtain the solution.

This paper considers hypersonic viscous gas flow over a plane finite delta wing at zero angle of attack with constant surface temperature, and a specified pressure on the wing trailing edge. The hypersonic interaction parameter has $\chi \geq O(1)$ and body surface is not cold. According to common assessments for the hypersonic boundary layer at $\chi \geq O(1)$, dimensionless coordinates and asymptotic representation for the flow functions are introduced. Substitution of these variables into the equations and execution of limiting transition $M_\infty \to \infty$, $Re_o \to \infty$ at $\chi \approx M_\infty^2 \tau^2 \geq O(1)$ result in three-dimensional boundary layer equations, where τ is characteristic dimensionless thickness of the boundary layer. It should be noted that the external edge of the wing boundary layer in hypersonic flow has been defined exactly in the first approximation as the shock layer where the gas density is a factor of τ^2 greater (in order of magnitude) than that of the boundary layer. To solve the system of equations for the boundary-layer pressure distribution, it is important to note that the pressure is not specified and is evaluated using interaction concepts; the external inviscid flow equations are obtained using hypersonic small-disturbance theory. The present paper considers flow over a delta wing with aspect ratio of $S = O(1)$. Thus strip theory is valid for the external inviscid flow with an accuracy of $O(\tau^2/S^2)$. An approximate tangential wedge formulae can be used to define the pressure and in dimensionless variables has the form

$$P = \frac{1}{\sigma\chi_*^2} + \frac{\sigma+1}{4}\left(\frac{\partial\delta e}{\partial x}\right)^2 + \frac{\partial\delta e}{\partial x}\left[\frac{1}{\chi_*^2} + \left(\frac{\sigma+1}{4}\cdot\frac{\partial\delta e}{\partial x}\right)^2\right]^{1/2};$$

$$\delta_e = \frac{\sigma-1}{2\sigma p}\int_0^\infty (g - u^2 - w^2)\,d\lambda,$$

where $\chi_* = M_\infty \tau$ and $\chi_*^2 = \chi S^{1/2}$. To find the unique for the boundary-value problem, it is necessary to specify the pressure distribution along the trailing edge of the wing.

In the vicinity of the top of the delta wing and near the leading edges, the hypersonic interaction parameter is calculated from the length of the region considered and is equal to $\chi_X = M_\infty^2 Re_X^{-1/2} \gg 1$; thus a strong viscous interaction is realized over these regions. To account for this, new variables, including the features of flow function behavior in these regions (Dudin, 1983), are introduced for the vicinity of the delta wing top and leading-edges. A system of equations describing a three-dimensional boundary layer on a plane delta wing with given pressure distribution $P_k(Z)$ at the wing trailing edge in a viscous interaction region is obtained by the transformations. The terms of the system of equations containing the longitudinal coordinate x drop out on the top of the delta wing top, and the boundary value problem is found to be dependent solely upon two independent variables, with the system obtained also describing flow over a semi-infinite plane delta wing in the strong interaction regime (Dudin, 1978). The system reduces to one involving the usual differential equations at the leading edges of the wing at the quantities $Z = \pm 1$. To solve the boundary value problem, it is necessary first to solve the equations at the leading edges; these solutions are used as boundary conditions to solve the equations that depend on two variables on th etop of the wing. Finally, the system of equations for the three-dimensional boundary layer is solved including specified boundary conditions at wing trailing edges $P_k(Z)$, and the solution obtained for the top of the wing and the leading edges.

To solve the equations, a finite-difference method is used that has been described by Dudin (1983). Derivatives in x- and z- coordinates are approximated with regard to the sign of their coefficients. Systems or difference equations for the u, w and g-functions are solved by the method of scalar sweep. To account for the disturbances being transmitted upstream to approximate pressure gradient through the longitudinal coordinate at $x > 0$, central differences are used. At the last layer where $x = 1$, the pressure gradient is not known and is selected in the solution of total boundary-value problem from the condition that the pressure distribution on the trailing edge obtained as a result of calculations is equal to a specified distribution. As an example, a flow over a plane delta wing is considered with the pressure at the trailing edge identified with a flow over semi-infinite delta wing in strong viscous interaction at $x = 1$. Thus $P_k(Z)$ was taken to be equal to the magnitude of the pressure obtained by solving the system of equations, describing delta wing top flow at $x = 0$. It was suggested that the numerical calculations $s = 1$ (the sweep angle was equal to 45°), $\gamma = 1.4$, $\sigma = 0.71$, $g_w = 0.05$, 0.1, 0.2 and $\chi_* = 1, 2, 5, 10^2, 10^5$.

Figure 1 presents calculated results for dimensionless pressure values P (solid lines) and coefficients of the skin friction in the longitudinal direction $\tau_w = (\partial u/\partial y)_w$ (the dashed lines) along x-axis in the plane of symmetry $Z = 0$ at $g_w = 0.05$ and the interaction parameter $\chi_* = 1, 2, \infty$ to which the curves 1 to 3 correspond.

Figure 2 presents the pressure distribution P (solid lines) and boundary layer displacement thickness δ_e (dashed lines) along the wing span at the longitudinal coordinate value $x = 0.5$. Dotted and dashed lines denote the specified pressure distribution $P_k(Z)$ on the trailing edge. It should be noted an essential change of boundary layer characteristics occurs depending on magnitude of the χ_* parameter. For this case and also in strong interaction regime (Dudin, 1983), the trailing edge effect is extended upstream by approximately 30% to 40% of the wing chord. An essential decrease in the pressure P in the vicinity of $x = 1$ at $\chi_* = 1$ leads to flow acceleration in the longitudinal direction and an increasing skin friction coefficient τ_w. The heat flux is changed in a similar way. It should be noted that the boundary layer displacement thickness decreases, essentially transferring from a strong viscous interaction to a moderate one. As a result, the pressure gradient over the wing span decreases except in the vicinity of the symmetry plane. In all cases considered, smooth convergence of the flow to the wing symmetry plane was realized. Flow characteristic calculations at the parameter $\chi_* = 2$ and $g_w = 0.1$, 0.2 (curves 5, 5, respectively) show that an increase in temperature coefficient significantly affects the flow parameters. Total aerodynamic characteristics as a function of interaction parameter magnitude were calculated. It has been shown that significant increase of these characteristics (by a factor of 1.6 to 1.8) occurs with a decrease of the parameter from 5 to 1 when flows with $\chi_* > 10$ are considered; the aerodynamic characteristics are not changed and coincide in practice with those corresponding to a strong viscous interaction.

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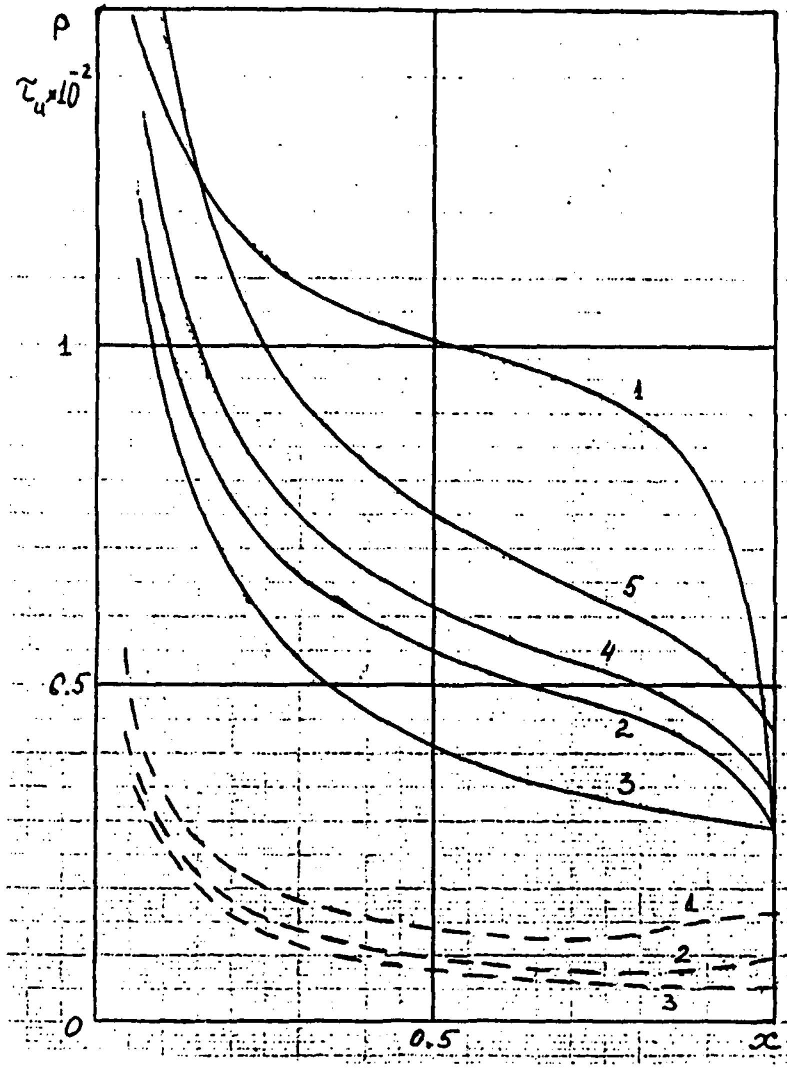
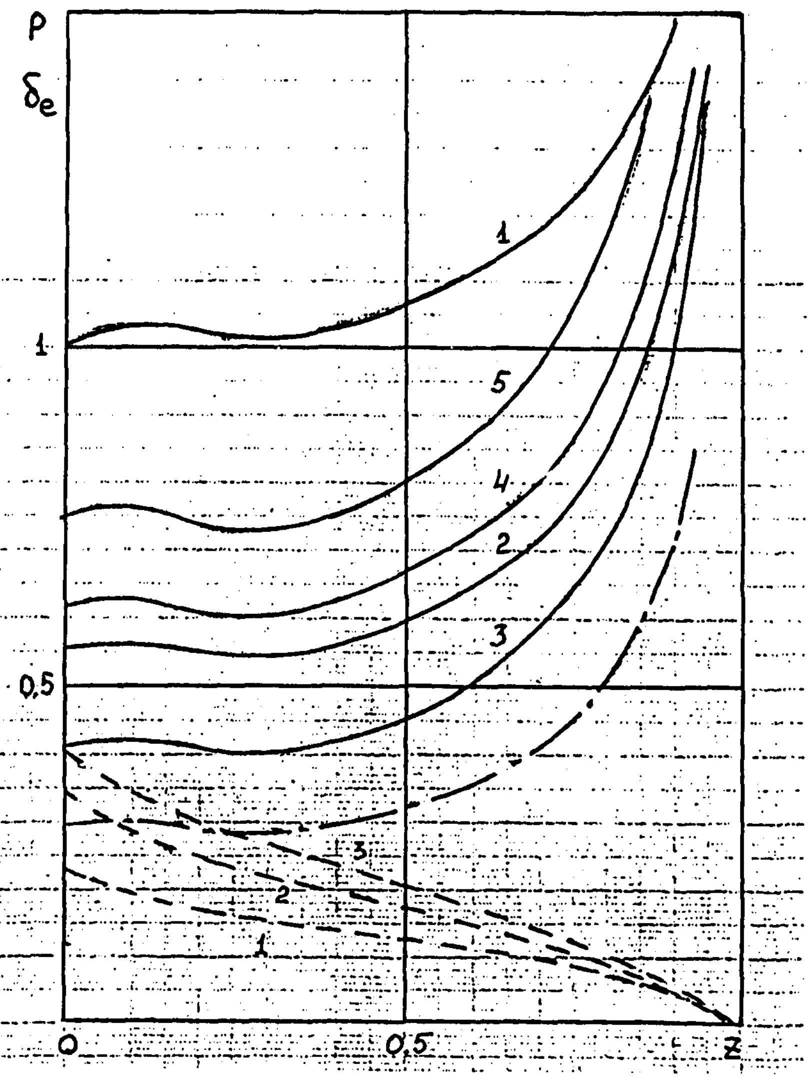


фиг.1



фиг.2

THREE-DIMENSIONAL BREAKAWAY MODEL OF A FREE-STREAMLINE TYPE

by

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The three-dimensional high-Reynolds-number laminar incompressible breakaway of a free streamline from a smooth surface is considered. The free-streamline model adopted in the inviscid flow region assumes that the flow potential satisfies Laplace's equation, the condition of zero normal velocity on the body surface and a condition of constant pressure on the free streamline downstream of the breakaway. The shape of the free streamline is fixed by the condition which requires that the tangent to the free streamline be parallel to the local velocity. The model also postulates that the velocity vector be continuous at the breakaway point. Since the analysis is confined to the vicinity of the breakaway point, the conditions at the infinity of the fluid are not considered.

As the leading term in the velocity expansion at breakaway is constant and equal to the velocity at the free streamline, a linearized version of the above problem applies to the second order solution in the neighborhood of the breakaway point. In order to solve this problem, we make use of Maz'ya-Plamenevsky's (1978) procedure and Kondrat'ev's theorem (1967), which establish that the solution may be expanded in the form of a series of local eigenfunctions and adjoint eigenfunctions. The inhomogeneous and non-linear terms contribute to the third- and fourth-order solutions. Hence, the tangential, the transverse, and the normal velocities at breakaway are

$$u = 1 + b\rho^{1/2} \sin(\phi/2) - \varkappa_z y - \frac{5}{6} b_2 x + A_1 z + 0(r^{3/2}),$$

$$w = b\rho^{1/2} \cos(\phi/2) - \varkappa_z^x y + \frac{5}{6} b^2 z + A_1 x - A_2 z + 0(r^{3/2}),$$

$$v = -\varkappa_x x - \varkappa_z^x z + A_2 y + 0(r^{3/2}),$$

where $\rho = (x^2 + z^2)^{1/2}$, $\tan(\phi) = z/x$, $0 \le \phi \le \pi$ and $r^2 = \rho^2 + y^2$; the body near the breakaway point 0 occupies the surface $y = -\varkappa_x x^2/2 - \varkappa_z^x zx - \varkappa_z z^2/2 + o(\rho^2)$. The x-axis of the Cartesian co-ordinates (x, y, z) is aligned with the flow at 0. The lengths x, y, z and the corresponding velocity components u, v, w are non-dimensionalized with respect to a typical body dimension and the free-stream velocity U_∞ respectively. The pressure is written as $p\rho_0 U_\infty^2 + p_\infty$, where ρ_0 is the fluid density and p_∞ is the pressure on the free streamline.

The most striking feature of the above solution is that, in general, two free streamlines stem from the breakaway point (according to either upper or lower choice of signs in the formulae below):

$$z = \pm\frac{2}{3} bx^{3/2} + A_1 \frac{x^2}{2} + O(x^{5/2}), \quad y = -\varkappa_x \frac{x^2}{2} \mp \frac{4}{15} æ_z^x bx^{5/2} + O(x^3).$$

Here A_1 and A_2 are arbitrary constants dependent on the overall flow properties. The constant b is also not known in advance. However, when $b < 0$, the pressure gradient is infinitely favorable and, since the boundary layer must remain attached at 0, this case is irrelevant to the phenomenon of breakaway. When $b > 0$, the pressure gradient is infinitely adverse and the boundary layer would have separated upstream of O. So, the case $b = 0$ appears to be the most physically realistic one.

In order to obtain a self-consistent account of the three-dimensional viscous separation involved here, we follow the approach adopted by Sychev (1972) for the two-dimensional problem and set $0 \leq b = 2c_1\epsilon(Re)$ with $c_1 \geq 0$ and $\epsilon(Re \to \infty) \to +0$. The structure of the flow at the breakaway then takes on the form of smooth separation at $Re \equiv \infty$; the adjustment of the viscous flow near separation is accomplished by means of the triple deck. In spite of obvious differences between the solution (1), (2) and the Kirchhoff model, the parallel arguments lead us to the conclusion that $\epsilon = Re^{-1/16}$. The streamwise extent of the triple-deck is $0(\epsilon^6)$, whereas the normal thicknesses of the decks are respectively $0(\epsilon^{10})$, $0(\epsilon^8)$, $0(\epsilon^6)$. The flow is then governed by the lower-deck properties where the usual triple-deck equations apply:

$$u_*\frac{\partial u_*}{\partial x_*} + w_*\frac{\partial u_*}{\partial z_*} + v_*\frac{\partial u_*}{\partial y_*} = -\frac{\partial p_*}{\partial x_*} + \frac{\partial^2 u_*}{\partial y_*^2},$$

$$u_*\frac{\partial w_*}{\partial x_*} + w_*\frac{\partial w_*}{\partial z_*} + v_*\frac{\partial w_*}{\partial y_*} = -\frac{\partial p_*}{\partial z_*} + \frac{\partial^2 w_*}{\partial y_*^2},$$

$$\frac{\partial u_*}{\partial x_*} + \frac{\partial w_*}{\partial z_*} + \frac{\partial v_*}{\partial y_*} = 0,$$

$$(\partial p/\partial x_*, \partial p/\partial z^*) = \frac{1}{2\pi}\int_{R_2}\int (\partial/\partial x, \partial/\partial z)\,\frac{\partial V_\infty}{\partial x}\,\frac{dxdz}{\sqrt{(x-x_*)^2 + (z-z_*)^2}}\,.$$

The boundary conditions are

$$u_* = w_* = v_*|_{y_* = 0} = 0\,,$$

to satisfy the no-slip condition at the body surface and

$$u_*(y_* \to \infty) = a_o y_*, \quad w_*(y_* \to \infty) = c_0 y_*,$$

$$\partial^2 v_*/\partial^2 x_*(y_* \to \infty) = (a_0\partial/\partial x_* + c_0\partial/\partial z_*)\partial\, V_\infty/\partial x_* \cdot y_*$$

and as $\rho_x \to \infty$

$$u_* = a_o y_* + \rho_*^{1/6} f_\epsilon(y_*/\rho_*^{1/3}), \quad w_* = c_0 y_* + \rho_*^{1/6}\, h_\epsilon(y_*/\rho_*^{1/3}),$$

$$(\partial p/\partial x_*, \partial p/\partial z_*) \to c_1\rho_*^{-1/2}(\sin(\phi/2), \cos(\phi/1))$$

to match with the main deck and the surrounding flow. Here $\rho_* = \rho/\epsilon^6$, $y_* = y/\epsilon^{10} a_0$, c_0

are constants and f_ϵ, h_ϵ are the correspondent solutions in the surrounding boundary layer.

Hence there is no formal difficulty about setting up the self-consistent procedure which accounts for the structure of the proposed three-dimensional breakaway model (1), (2), provided that the above system is solvable for $c_1 > 0$. We conclude then that apart from the classical breakaway model based on Kirchoff's solution (which in the three-dimensional case would imply the presence of a freestream surface stemming from a line of separation) the three-dimensional breakaway scenario can also be based on a free-streamline model (1), (2). The latter is characterized by the breakaway of a free streamline (2) at one point of the body surface, which is fixed by the condition of "smooth separation" b=0.

SEPARATION PHENOMENON IN A HYPERSONIC FLOW WITH STRONG WALL COOLING: SUBCRITICAL REGIME

by

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This paper is concerned with separation phenomenon that occur in a hypersonic flow past a curved wall which is cooled. An interactive boundary layer approach is used within the weak global interaction regime, where the parameter $\chi \sim M_\infty^2 Re^{-1/2}$ is much smaller than unity. As established by Neiland (1973), in this case the induced pressure obeys the interaction law

$$P = sign\,(L)\,\frac{dP}{dX} + N^{-1}\,\theta + \frac{dF}{dX}, \tag{1}$$

where X is the scaled streamwise variable in the interaction region, θ is the slope of the streamlines at the outer edge of viscous sublayer, F is the scaled form of the body surface,

$$L = \int_0^\sigma (M^{-2} - 1)\,dY, \tag{2}$$

M being the Mach number distribution across the boundary layer. Here a curved surface having a shape defined by $F = \theta\left(X + \sqrt{X^2 + 1}\right)$, corresponding to a smooth compression ramp, will be addressed. If $L < 0$, the character of interaction is analogous to a flow with a supersonic mainstream, while for $L > 0$, the nature of the interaction is similar to that with a subsonic mainstream. The case $L < 0$ has been studied by Kerimbekov, Ruban and Walker (1992) and this study paper is concerned with regime $L > 0$.

The parameter N is proportional to the ratio of displacement thickness variation in the main part of the boundary layer to the thickness of a viscous sublayer immediately adjacent to the surface. For certain ranges of the Mach number, Reynolds number, and the wall temperature, the parameter N becomes large with respect to unity; thus the main part of the boundary layer gives the dominant contribution to displacement thickness. In this situation a classical approach may be used to consider the evolution of the viscous lower deck (i.e. the sublayer). This approach remains valid until θ is smaller than some critical value θ_c. At $\theta = \theta_c$ the flow is in the forward direction everywhere, but the skin friction vanishes linearly at a single point denoted by X_s. This behavior of the skin friction at $\theta = \theta_c$ leads to the emergence of an inner interaction region in the vicinity of X_s having a streamwise scaling $|X - Xs| = O(N^{-2/3})$. Marginal separation theory (Ruban, 1982; Stewartson, Smith, and Kaups, 1982) may be applied to describe the interaction in this region. According to this theory, the stream function in the lower deck of inner interaction region can be expressed in the form

$$\psi = \tfrac{1}{2}N^{-1/2}\,\lambda_1 y^3 + \tfrac{1}{2}N^{-1}\,\lambda_1 y^2 A(x) + \ldots + N^{-3/2}\tilde{\Psi} + \ldots, \tag{3}$$

where y and x are appropriate scaled transverse and streamwise variables respectively, λ_1 is related to the value of the pressure gradient at $X = X_s$ and $\theta = \theta_c$, and A(x) is proportional to both the skin friction and the boundary layer displacement thickness. It is shown that a solution to the problem for $\bar{\Psi}$ exists provided A(x) satisfies a solvability condition

$$A^2(x) - x^2 + 2\alpha = -\int_{-\infty}^{x} \frac{A'(\xi)+1}{\sqrt{x-\xi}}\, d\xi, \tag{4}$$

where α is proportional to the scaled departure of the θ from its critical value θc. Numerical solutions of the integro-differential "fundamental" equation (4) for A(x) are obtained. Calculated results and an analytical solution calculated for the limiting process $\alpha \to 0^-$ show that marginal separation theory becomes invalid in the neighborhood of the reattachment point for small enough α. Since the coordinate of the reattachment point x_r tends to infinity as α approaches zero through negative values, reconsideration of the reattachment problem becomes necessary when x_r is proportional to $N^{1/3}$. Formulation of the problem for nonlinear reattachment region having a streamwise scaling $\Delta x = O(N^{-2/3})$ is reduced to the consideration of the nature of the solution in the lower deck. The governing parameter for this problem is β, which is defined by

$$x_r = \beta N^{1/3}, \tag{5}$$

for large values of x_r. Numerical solutions of the problem for $\beta < 1.225$ indicates that a reversed-flow singularity is encountered in the interacting-boundary-layer problem under consideration, with the critical value of β being nearly 1.23. This type of singularity seems to be predicted by Smith (1988) as a local breakdown of any interactive boundary-layer solution at a finite value of the controlling parameter.

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NONLINEAR INSTABILITY OF SUPERSONIC VORTEX SHEETS AND SHEAR LAYERS

by

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According to the linear theory, a vortex sheet in supersonic flow is neutrally stable if the Mach number is large enough, whereas a thin shear layer can be unstable to small disturbances, with growth rate approaching zero in the long-wave limit. In this sense, the shear layer approaches a vortex sheet in the limit of vanishing thickness. A corresponding consistency has not yet been demonstrated for the slow nonlinear response to small disturbances. For a vortex sheet, Artola and Majda (1987) have demonstrated weakly nonlinear instability, in the context of resonant response to a sound wave at oblique incidence. A steepening of compression waves outside the sheet leads to distortion of the shape of the sheet, such that corners develop with a weak shock wave on one side and a weak centered expansion on the other. A long-wave limit for thin shear layers has been considered by Balsa (1991, 1993), such that the ratio of shear-layer thickness to disturbance wavelength is small, and the ratio of disturbance amplitude to shear-layer thickness is likewise small, the two ratios being of the same order of magnitude. Viscous effects and nonlinearity enter the equations for the critical layer, but nonlinear effects do not influence the external flow at this stage, so the solution obtained in this limit does not appear capable of approaching the vortex-sheet solution if the thickness is decreased still further.

It appears that another limit must be considered, where the disturbance amplitude and shear-layer thickness are of the same order of magnitude. The small parameter is then the ratio of one of these lengths to the disturbance wavelength. This limit is considered here, again with viscosity chosen to balance nonlinearity in a suitable sense. The coordinate system is chosen to move at one of the speeds found for linear neutral disturbances to a vortex sheet, so that the flow is nearly steady. In the external flow, a periodic initial disturbance is prescribed, with a finiteness requirement in the second approximation giving results equivalent to those of Artola and Majda. Shock waves initially are present only at large distances, and move either toward or away from the shear layer as the mean surface becomes increasingly distorted. Perturbations in most of the shear layer are described by a slightly modified version of Balsa's derivation. The equivalent second-order surface, with shear layer displacement effects taken into account, is now different when viewed from above or below, and matching with the external flow is modified. The velocity jump found at the critical layer is required to agree with that obtained from a critical-layer solution. The critical layer in this limit can be characterized as an equilibrium inviscid critical layer, somewhat like that described by Goldstein and Hultgren (1988) for an incompressible shear layer. In the present case, in a first approximation, the temperature in the critical layer is constant along a streamline, and the vorticity changes because of a baroclinic forcing. The dependence on a stream function is found from a solvability condition for a later approximation, somewhat like that of Goldstein and Hultgren. Suitable asymptotic matching of solutions in the three regions then leads to a description of the slow increase in mean-surface distortion. As the ratio of amplitude to thickness increases or decreases, the results approach those for a vortex sheet match with a later stage of Balsa's long-wave formulation.

CALCULATION OF SEPARATED FLOW ON A CIRCULAR CONE WITH VISCOUS-INVISCID INTERACTION

by

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One of the main problems in the calculation of separated flow near a smooth body at high Reynolds numbers is determinating separation line location. This problem may be solved either by using empirical data or by carrying out an interactive procedure using a composite solution of the equations describing inviscid global separated flow and boundary layer equations near smooth surface (Gaifullin and Zakharov, 1990). Zakharov (1976) determined a family of inviscid solutions for symmetric separated flow near a slender circular cone at fixed incidence. The purpose of this paper is to demonstrate how to select the single solution from this family for each Reynolds number and hence determine the calculation of separation line location on the cone.

It follows from Laplace equation that the adverse pressure gradient upstream of separation is infinite along the inviscid separation line (i.e. the vortex sheet shedding line). This is not the case when smooth condition is valid on inviscid separation line: the curvature of vortex sheet on shedding line is finite and is equal to that of body surface.

Consider a symmetric fluid flow with U_∞ over circular cone with semi-apex $\theta \ll 1$ with an angle of incidence $\alpha \sim \theta$. Under these assumptions slender body theory is valid. The case of circular cone is a self-similar involving separated flow over a circle that is expanding with constant rate. When separation from a slender cone takes place, the disturbed flow is conical to a first approximation (except in the vicinity of the apex and bottom), and this is confirmed by the straightness of the primary separation lines.

A simplified mathematical model is used in this paper to describe flow separation over the cone, with two symmetric vortex sheets shedding from the primary separation lines. The inviscid method involves introducing vortex sheet which is asymptotic in the vicinity of separation lines. For the vortex sheet core the well-known model "vortex-feeding cut" is used. Continuous vortex sheets are simulated by large numbers (usually 40 and more) of discrete vortices. At distances less than discretization step, vortex panels are used to calculate the induced velocities. In the symmetric case corresponding to the plane self-similar problem, there is the only main parameter, namely the relative value of incidence $\overline{\alpha} = \alpha/\theta$.

Let us confine ourselves to laminar boundary layer separation from the cone. For the boundary layer problem, an orthogonal curvilinear co-ordinate system was adopted in which the cone surface is denoted by $\zeta = 0$; here ζ is the distance normal to the surface, ξ is the distance along the cone generators from the apex, and η is the angle between any generator of the cone and the lowest generator, measured in the cross-plane. The boundary-layer equations for incompressible flow in this system are

$$u\frac{\partial u}{\partial \xi} + v\frac{\partial u}{\partial \zeta} + \frac{w}{r}\frac{\partial u}{\partial \eta} - \frac{w^2}{r}\sin\theta = \nu\frac{\partial^2 u}{\partial \xi^2} \quad (1)$$

$$\frac{\partial p}{\partial \zeta} = 0 \tag{2}$$

$$u\frac{\partial w}{\partial \xi} + v\frac{\partial w}{\partial \zeta} + \frac{w}{r}\frac{\partial w}{\partial \eta} + \frac{wu}{r}\sin\theta = -\frac{1}{r}\frac{\partial p}{\partial \eta} + \nu\frac{\partial^2 w}{\partial \zeta^2} \tag{3}$$

$$\frac{\partial(ur)}{\partial \xi} + r\frac{\partial v}{\partial \zeta} + \frac{\partial w}{\partial \eta} = 0\,, \tag{4}$$

where $r(\xi)$ is the radius measured in the cross-plane of the cone, (u, v, w) are the velocity components in the (ξ, ζ, η) directions, p is the pressure and ρ is the density. The boundary conditions are

$$u = v = w = 0 \qquad \text{when} \qquad \zeta = 0 \tag{5}$$

$$u = U_e(\eta), \quad w = w_e(\eta) \qquad \text{when} \qquad \zeta = \delta(\eta) \tag{6}$$

where $\delta(\eta)$ is the boundary layer thickness.

It is possible to find solutions in the which the velocity components are functions of $\lambda = \zeta(u_e/\xi\nu)^{1/2}$ and η only according to

$$u = U_e(\eta)E(\eta,\lambda)$$

$$w = W_e(\eta)G(\eta,\lambda)$$

$$v = \left(\frac{U_e\nu}{\xi}\right)^{1/2}\left(V(\eta,\lambda) + \frac{1}{2}\lambda E - \frac{U_e' W_e}{2u_*^2\sin^2\theta}\lambda G\right).$$

Under the assumptions of slender body theory, it can be shown that equations (1) – (4) reduce to

$$\frac{\partial^2 E}{\partial \lambda^2} = V\frac{\partial E}{\partial \lambda} + BG\frac{\partial E}{\partial \eta} \tag{7}$$

$$\frac{\partial^2 G}{\partial \lambda^2} = V\frac{\partial G}{\partial \lambda} + C(G^2 - 1) + EG - 1 + BG\frac{\partial G}{\partial \eta} \tag{8}$$

$$\frac{\partial V}{\partial \lambda} = -\frac{3}{2}E - CG - B\frac{\partial G}{\partial \eta}\,, \tag{9}$$

where $B(\eta) = W_e(\eta)/(U_\infty \sin\theta)$, $C(\eta) = W_e'(\eta)/(U_\infty \sin\theta)$. The boundary conditions now become

$$E = V = G = 0 \qquad \text{when} \qquad \lambda = 0 \tag{10}$$

$$E \to 1,\ G \to 1 \qquad \text{when} \qquad \lambda \to \infty. \tag{11}$$

The system of equations (7) – (11) was solved numerically using finite-difference relations which are second order accurate. In the numerical solution the flow region from $\eta = 0$ to $\eta = \pi$ was divided on three parts as follows:

(a) From $\eta = 0$ to the point of boundary-layer separation at $\eta = \eta_1$ and from $\eta = \eta_1$ to any point $\eta = \eta_2$.

Here we think of boundary-layer separation line as a line on which the normal skin friction component is equal to zero. The region (η_1, η_2) must be greater than the length of interaction. The governing equations in this region are equations (7) – (9) if $G(\lambda) \geq 0$ and

$$\frac{\partial^2 E}{\partial \lambda^2} = V \frac{\partial E}{\partial \lambda}$$

$$\frac{\partial^2 G}{\partial \lambda^2} = V \frac{\partial G}{\partial \lambda} + EG - 1 - C$$

$$\frac{\partial V}{\partial \lambda} = -\frac{3}{2} E$$

if $G(\lambda) < 0$. The strength of source in region $(0, \eta_2)$ is

$$q(\eta) = 2 \frac{V(\eta, \lambda) + \lambda \left(\frac{3}{2} + C\right)}{\theta Re^{1/2}} \qquad \text{when} \qquad \lambda \to \infty. \tag{12}$$

(b) From the point $\eta = \eta_2$ to any point $\eta = \eta_3$.

The governing equations in this region are inviscid. The region (η_2, η_3) is small and equation of vortex sheet here is $\zeta = k\alpha^{2/2}$, where $\alpha = r(\eta - \eta_o)$ is the distance along the cone from the vortex sheet separation point. The vortex sheet in this region was represented by a source with strength

$$q(\eta) = \frac{3}{2} \frac{\gamma k}{\pi} \alpha^{2/3}, \qquad \eta_2 \leq \eta \leq \eta_3 \tag{13}$$

and sink $\frac{\lambda k}{\pi} \alpha^{3/2}(\eta_3)$ at the point $\eta = \eta_3$. Here γ is the vortex sheet strength.

(c) From point $\eta = \eta_3$ to $\eta = \pi$.

The governing equations in this region are inviscid. The velocity in the η direction may be expressed in the form

$$W_e(\eta) = W_{eo}(\eta) + W_{eq}(\eta) \tag{14}$$

where $W_{eq}(\eta)$ is the velocity induced by the sources (12), and $W_{eo}(\eta)$ is a function of U_∞, η_o, η_2. Iterations must be used to obtain the solution for velocity profiles, boundary velocity $W_e(\eta)$, strength of sources $q(\eta)$, and the vortex sheet separation point

η_o. The iteration process is repeated until the source strength (12) and source strength (13) is equal at the point η_2.

The last formula (14) is formally identical to the Veldman's formula (Veldman, 1981), but in this work W_{eo} is not constant in the iteration process, because η_o is unknown. Figure 1 shows the strength of sources derived from equations (12) and (13). Figure 2 gives comparison between the present solutions and results of Smith (1982).

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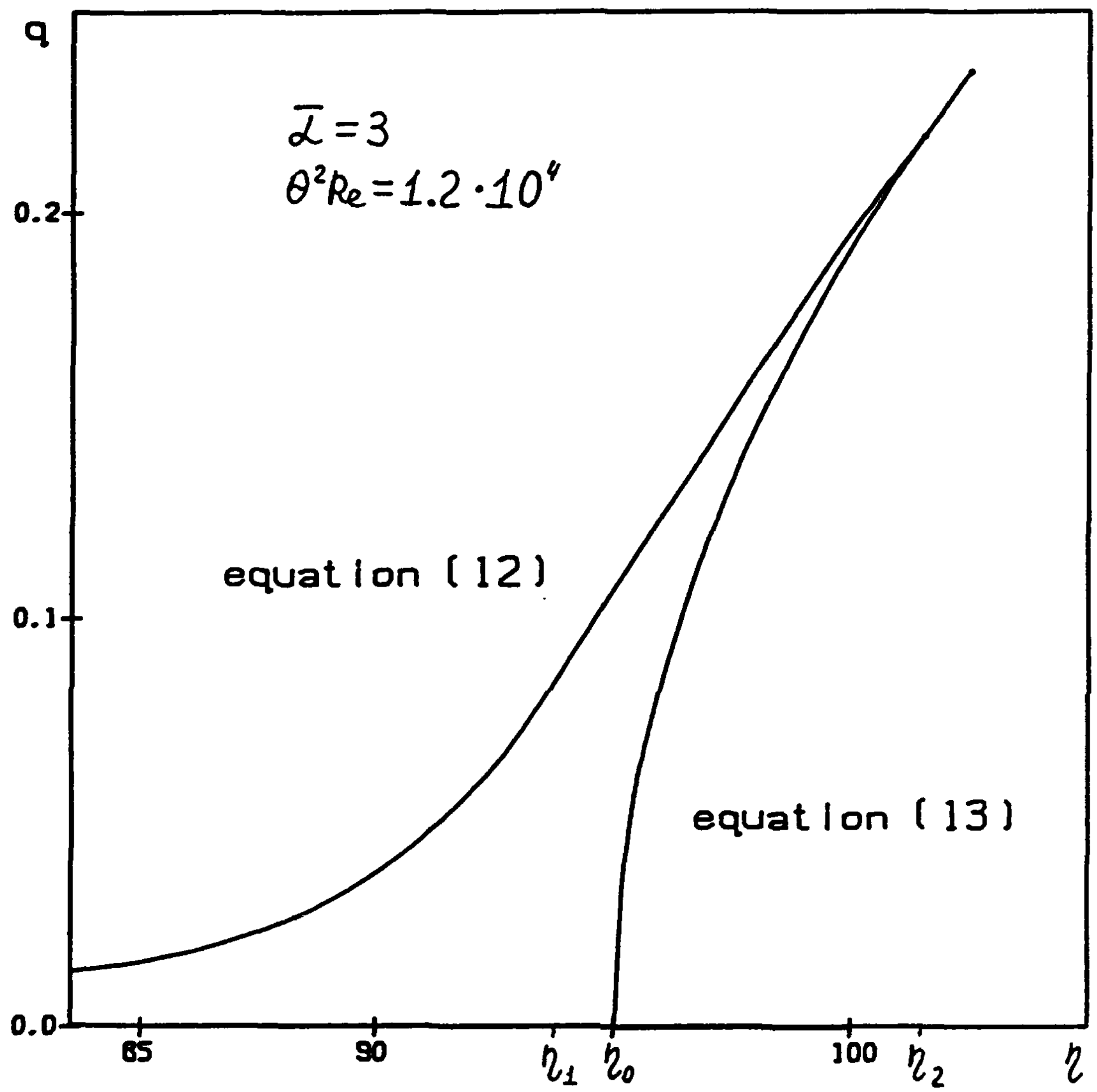
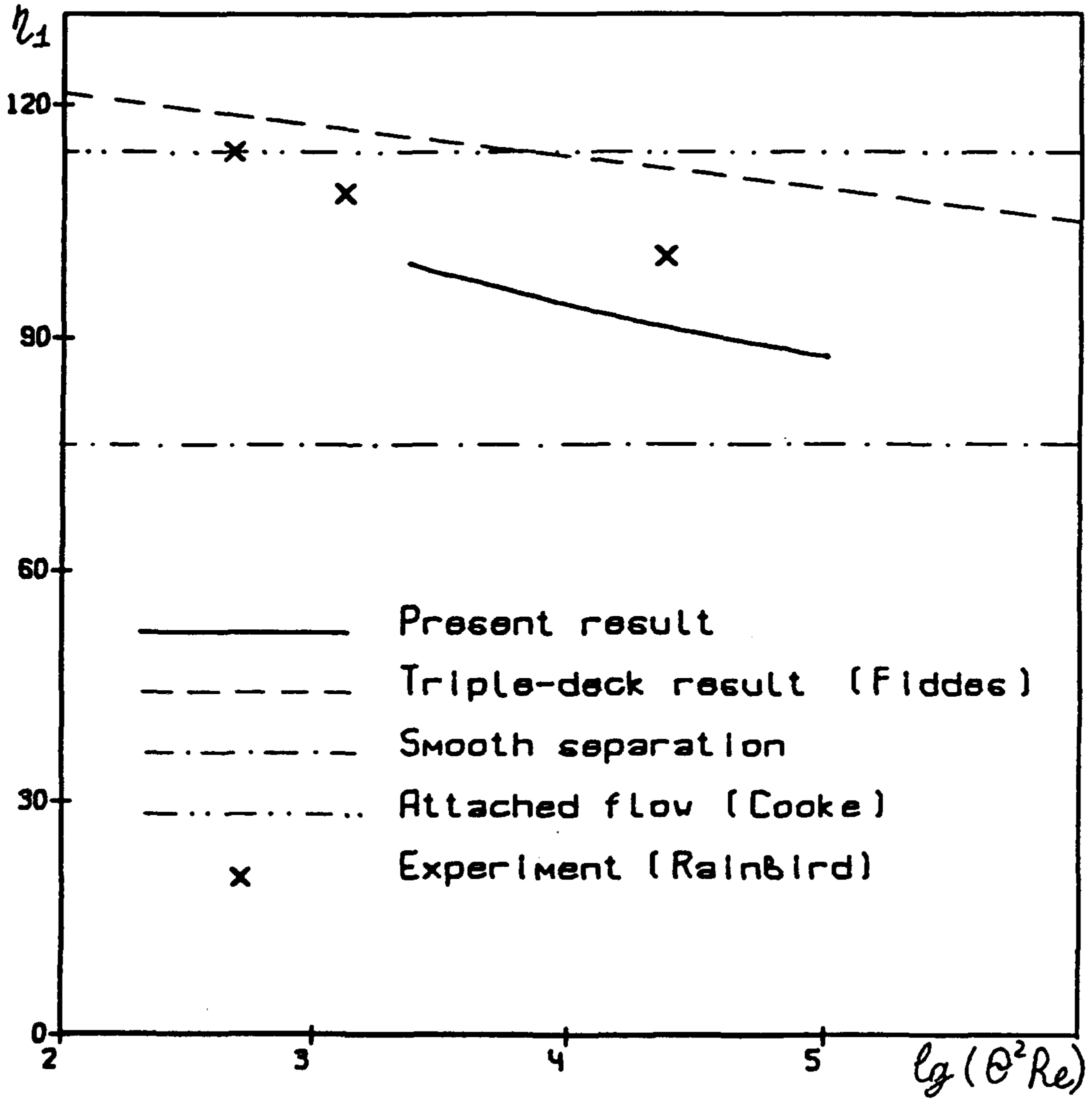

q
$\bar{\mathcal{L}}=3$
$\theta^2 Re=1.2\cdot10^4$
0.2
0.1
0.0
equation (12)
equation (13)
85
90
η_1
η_0
100
η_2
η

Figure 1



η_1
120
90
60
30
0
2
3
4
5
$lg(\theta^2 Re)$
Present result
Triple-deck result (Fiddes)
Smooth separation
Attached flow (Cooke)
Experiment (Rainbird)

Figure 2

UNSTEADY HYPERSONIC THIN SHOCK LAYERS AND FLOW STABILITY

by

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An asymptotic theory of unsteady shock layers has been developed. This formulation is a generalization of the steady case treated by Cole (1957). Major aspects of the analysis are the use of a "Newtonian" distinguished limit embedded within Hypersonic Small Disturbance Theory (HSDT) (Van Dyke, 1954). IF M_∞ is the freestream Mach number and δ is a characteristic flow deflection, then a hypersonic similarity parameter $H \equiv 1/M_\infty^2\delta^2$ governs the HSDT flow which occurs in the distinguished limit $\delta \rightarrow 0$, H fixed. To keep the flow in view in the HSDT limit, a strained version of the transverse coordinate $\tilde{y}$ to the flow y, where $\tilde{y} \equiv y/\delta$ is used. If the characteristic time is of the same order as the freestream convection speed, then the convective operators can be readily generalized from steady to unsteady flow, by the addition of a $\alpha/\alpha t$ term in the HSDT approximation of the substantial derivatives. Treatment of thin shock layers then proceeds analogous to the procedure given in Cole (1957) involving the use of coordinates measured from the body surface, as well as the transverse velocity ν measured from its value on the body. If γ is the specific heat ratio, a Newtonian limit defined in terms of the Newtonian similarity parameter $N \equiv H/\lambda$, where $\lambda \equiv \gamma - 1/\gamma + 1$, is used to study high Mach number flows as the shock approaches the body. The distinguished limit involves $\lambda \rightarrow 0$ for N and y^* fixed where $y^* \equiv (\tilde{y} - \tilde{y}_{body})/\lambda$ as a "boundary layer" coordinate used to preserve the fine structure of the shock layer as $\gamma \rightarrow 1$. Substitution of the unsteady HSDT and Newtonian asymptotic expansions of the density, transverse velocity and pressure into the Euler equations gives initial-boundary value problems for the approximate flow quantities subject to shock conditions on a free boundary corresponding to a bow shock wave as well as a body flow tangency condition. If x and t are the streamwise coordinate and time respectively, introduction of characteristic coordinates $\xi = x + t/2$ and $\eta = x - t/2$ facilitates the unsteady Von Mises transformation $(x,y,t) \rightarrow (\xi, \eta, \psi)$. Integration of the energy equation in unsteady Von Mises variables shows that the entropy convects at constant speed downstream on the streamlines of the flow. Solution of the equations gives an unsteady generalization of the Newton-Busemann law and the complete flow field between the body and its shock. The generalized form gives the pressure distribution over a body whose shape changes with time. A major thrust is to use the Newtonian solution to study the spatial stability of finite disturbances occurring in strong interaction between a hypersonic boundary layer and shock layer. Similarity solutions relevant to this interaction and matching issues are discussed in relation to the study of small amplitude inviscid fluctuations (Malmuth, 1992).

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WAVE RESISTANCE OF A SUBMERGED BODY MOVING WITH AN OSCILLATING VELOCITY

by

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There are numerous papers treating stationary waves due to the forward motion of a body (see, for example, Wehausen, 1973, and Baar and Price, 1988, and the bibliography cited therein). Non-stationary ship waves are investigated to a lesser degree (see Sretenskiy, 1977, and Newman, 1978). The situation is opposite from the mathematical point of view. Some solvability and uniqueness theorems are proved for different formulations of the linear initial value problem describing forward motion of a submerged body (see Garipov, 1967, and Hamdache, 1984).

In this work the effect of high-frequency oscillations of forward velocity on the wave-making resistance is considered. Let a body be submerged in an incompressible inviscid heavy fluid which, for simplicity, is of infinite depth. Let its dimensionless velocity have the form $U(t/\epsilon)$, where $U(\tau)$ is a positive differentiable function having the unit period and t is the dimensionless time. Only dimensionless quantities will be used in the paper. It means that t is obtained by dividing the dimensional time by, for example, $(L/g)^{1/2}$, where g is the acceleration of gravity and L is the characteristic length. In the same manner U is obtained by dividing the dimensional velocity by $(Lg)^{1/2}$. If $\epsilon \ll 1$, then the velocity oscillates at the frequency, which is high in comparison with $(g/L)^{1/2}$. In this case singular perturbation methods are applicable.

Our aim is to derive the asymptotic formula for the wave-making resistance $R(t, \tau)$ of the body, $\tau = t/\epsilon$. Then we shall compare the mean value

$$<R> = \int_0^1 R \, d\tau$$

with the wave-making resistance $R_o(t)$ of the same body moving with the mean velocity $<U>$. Numerical computations for the 2D problem show that there exist cylinders such that $|<R>| < |R_o|$ (up to a term $O(\epsilon)$).

Formulation and Solution

Let a solid body occupy the domain $D \subset R^3_- = \{(x, y, z): y < 0, (x, z) \in R^2\}$ and D is bounded by the closed surface $s \subset R^3_-$. Let the plane $\{y = 0, (x, z) \in R^2\}$ is the free surface of fluid at rest. Thus the body is submerged, and it moves in the direction of the x-axis with the velocity $U(t/\epsilon)$. We choose the vertical size of the body as the characteristic length L.

We seek a pair (ϕ, ξ) satisfying

$$\left.\begin{aligned}
&\nabla^2\phi=0 \quad \text{in } W=R^3_-\backslash\overline{D} && (1)\\
&\phi_t-U\phi_x+\xi=0, \quad y=0 && (2)\\
&\eta_t-U\eta_x-\phi_y=0, \quad y=0 && (3)\\
&\partial\phi/\partial n=U\cos(n,x) \text{ on } S && (4)
\end{aligned}\right\} \quad t\geq 0$$

$$\left.\begin{aligned}
&\phi=f_o, \quad y=0 && (5)\\
&\eta=f_i, \quad y=0 && (6)
\end{aligned}\right\} \quad t=0$$

Thus $\phi(x, y, z, t, \epsilon)$ can be regarded as a velocity potential of induced waves in a coordinate system moving with the body, and $\eta(x, z, t, \epsilon)$ is the corresponding elevation of free surface. The unit normal $\vec{n}$ is directed into W.

For the pair (θ,η) satisfying (1)–(6) with $U(t/\epsilon)$ described above, the following asymptotic formulae are true as $\epsilon\to 0$:

$$\begin{aligned}\phi=&[U(t/\epsilon)-\langle U\rangle]\,[v_o(x, y, z)+\phi_o(x, y, z, t)]\\ &+\epsilon[\beta(t/\epsilon)\,v_1(x, y, z, t)+\phi_1(x, y, z, t)]+O(\epsilon^2)\end{aligned} \tag{7}$$

$$\begin{aligned}\eta=&\eta_o(x, z, t)+\epsilon\{\beta(t/\epsilon)\,[(\partial\eta_o/\partial x)(x, z, t)\\ &+(\partial v_o/\partial y)(x, 0, z)]+\eta_1(x, z, t)\}+O(\epsilon^2).\end{aligned} \tag{8}$$

Here

$$\beta(\tau)=\int_0^1 G(\tau, \sigma)\,U(\sigma)\,d\sigma, \quad 0\leq\tau\leq 1$$

$$G(\tau,\sigma)=H(\tau-\sigma)\;-\;(\tau-\sigma)\;-\;1/2,$$

where H is the Heaviside-function. The function $\beta(\tau)$ is extended periodically to the half-axis $\tau>1$. The function $G(\tau, \sigma)$ is a generalized Green function of the periodic boundary value problem for the operator $d/d\tau$.

The functions $v_m(m=0, 1)$ must be determined from

$$\begin{aligned}&\nabla^2 v_m=0 \text{ in } W,\; v_m=\delta_{1m}(\partial\phi_o/\partial x) \text{ for } y=0,\\ &\qquad\partial v_m/\partial n=\delta_{0m}\cos(n, x) \text{ on } S.\end{aligned} \tag{9}$$

The pairs $(\phi_m, \eta_m)(m=0, 1)$ are the solutions of the initial boundary value problems

$$\nabla^2\phi_m=0 \quad \text{in } W, \tag{10}$$

$$\left.\begin{aligned}
&\frac{\partial\phi_m}{\partial t} - <U> \frac{\partial\phi_m}{\partial x} + \eta_m = 0, \quad y = 0, && (11)\\
&\frac{\partial\eta_m}{\partial t} - <U> \frac{\partial\eta_m}{\partial x} - \frac{\partial\phi_m}{\partial y} = 0, \quad y = 0, && (12)
\end{aligned}\right\} \quad t \geq 0$$

$$\partial\phi_m/\partial n = \delta_{0m} <U> \cos(n, x) \text{ on } S, \tag{13}$$

and

$$\left.\begin{aligned}
&\phi_m = [-\beta(0)]^m \partial^m f_o/\partial x^m, \quad y = 0 && (14)\\
&\eta_m = [-\beta(0)]^m \left[\frac{\partial^m f_1}{\partial x^m} + \delta_{1m}\frac{\partial v_o}{\partial y}\right], \quad y = 0 && (15)
\end{aligned}\right\} \quad t = 0$$

Here δ_{km} is the Kronecker delta.

Thus ϕ_o can be regarded as the velocity potential of waves due to the body at the forward speed $<U>$, and η_o is the corresponding elevation of free surface. The formula (8) demonstrates that the free surface elevations η and η_o coincide up to a term $0(\epsilon)$, but this is not valid for the potentials ϕ and ϕ_o.

For $m = 0$ the problems (9) and (10) – (15) can be solved independently of each other. Then we have to find v_1 from (9), and at last we obtain the solution ϕ_1 of the problem (10) – (15). In an analogous manner, one can derive an arbitrary number of terms to extend the expansions (7) and (8).

<u>Wave resistance and other characteristics</u>

Applying the usual formula, we get from (7) that for the force $\vec{F}(t, \tau)$ acting on the body, the following asymptotics are true

$$\begin{aligned}
\vec{F} = {} & \frac{U'(\tau)}{\epsilon} \int_S v_o \vec{n}\, dS + \int_S \left(\frac{\partial\phi_o}{\partial t} - <U> \frac{\partial\phi_o}{\partial x}\right)\vec{n}\, dS \\
& - [U(\tau) - <U>]^2 \int_S \frac{\partial v_o}{\partial x}\vec{n}\, dS \\
& + [U(\tau) - <U> \int_S \left[v_1 - <U> \frac{\partial v_o}{\partial x} - \frac{\partial\phi_o}{\partial x}\right]\vec{n}\, dS + O(\epsilon).
\end{aligned}$$

Then, averaging in the variable τ, we find that

$$<\vec{F}> = \vec{F}_o(t) - (<U^2> - <U>^2) \int_S \frac{\partial v_o}{\partial x}\vec{n}\, dS + O(\epsilon), \tag{16}$$

where

$$\vec{\mathbf{F}}_o(t) = \int_S \left(\frac{\partial \phi_o}{\partial t} - <U> \frac{\partial \phi_o}{\partial x}\right) \vec{n}\, dS,$$

is the force acting on the body moving with the mean velocity $<U>$. The second term in equation (16) is proportional to the dispersion of velocity $<U^2> - <U>^2 \geq 0$.

Let us consider the horizontal component $R(t, \tau)$ of the force (wave-making resistance). Using the boundary value problem (9), we get from (16) that

$$<R> = R_o(t) - (<U^2> - <U>^2)\, I_o + O(\epsilon),$$

$$I_o = \int_S \frac{\partial v_o}{\partial x} \cos(n, x)\, dS = \frac{1}{2} \int_S |\nabla v_o|^2 \cos(n, x)\, dS, \tag{17}$$

$$R_o(t) = \int_S \left(\frac{\partial \phi_o}{\partial t} - <U> \frac{\partial \phi_o}{\partial x}\right) \cos(n, x)\, dS.$$

The last formula is the wave resistance of the body at the constant speed $<U>$. The asymptotic formula for R then may be written in the following form

$$R(t, \tau) = <R> + \frac{U'(\tau)}{\epsilon} \int_W |\nabla v_o|^2\, dxdydz$$

$$+ \left[<U> - U(\tau)\right] \int_S \left(\frac{\partial v_o}{\partial x}\frac{\partial \phi_o}{\partial n} + v_o \frac{\partial^2 \phi_o}{\partial x \partial n}\right) dS + O(\epsilon).$$

The supplied power can be obtained by multiplying $(-R)$ by U. Then the average supplied power is given by the following expression

$$-<U>R_o + (<U^2> - <U>^2 \left\{<U>\, I_o + \int_S \left(\frac{\partial v_o}{\partial x}\frac{\partial \phi_o}{\partial n} + v_o \frac{\partial^2 \phi_o}{\partial x \partial n}\right) dS\right\} + O(\epsilon),$$

which differs both from the power required for the motion at the mean speed $<U>$ and from the power required for overcoming of the mean wave resistance $<R>$.

Discussion and numerical examples

Formula (17) shows that the sign of the difference $<R> - R_o$ depends on the value of I_o. As the wave resistance is directed opposite to the x-axis, then we have the inequality $|<R>| > |R_o|$ if $I_o > 0$. So, it is of interest to find bodies with $I_o < 0$.

It is easy to see that $I_o = 0$ if the body is symmetric about the middle-plane (without loss of generality, we can choose $x = 0$ as the middle-plane). Indeed, in this case cos(n,x) is an odd function of x. Then the solution $v_o(x, y, z)$ of the boundary problem (9) is an odd function of x. The same is true for the integrand in I_o.

For a numerical example, the two-dimensional problem (9) with an isosceles triangle ABC (see Figure 1) as contour S is convenient. For any contour S we have

$$I_o = \int_S \left[\cos^3(n, x) + (\partial v_o/\partial s)\cos(s, x)\cos(n, x)\right] dS.$$

In the case of triangle ABC, we get

$$\int_{ABC} \cos^3(n, x)\, dS = -\sin^2\alpha,$$

$$\int_{ABC} (\partial v_o/\partial s)\cos(s, x)\cos(n, x)\, dS = \tfrac{1}{2}\left[2v_o(B) - v_o(A) - v_o(C)\right]\sin 2\alpha.$$

Thus, $I_o(ABC) < 0$ if $2v_o(B) - v_o(A) - v_o(C) < \tan\alpha$. For triangles that correspond to the points above the curve (see Figure 1), the inequality $I_o(ABC) < 0$ holds. The opposite inequality takes place for triangles that correspond to the points below the curve. For a triangle $AB'C$, which is symmetric about y-axis with any triangle ABC shown in Figure 1, the inequality $I_o(AB'C) > 0$ is valid.

Numerical results are also obtained for right triangles that have one of their legs on the y-axis.

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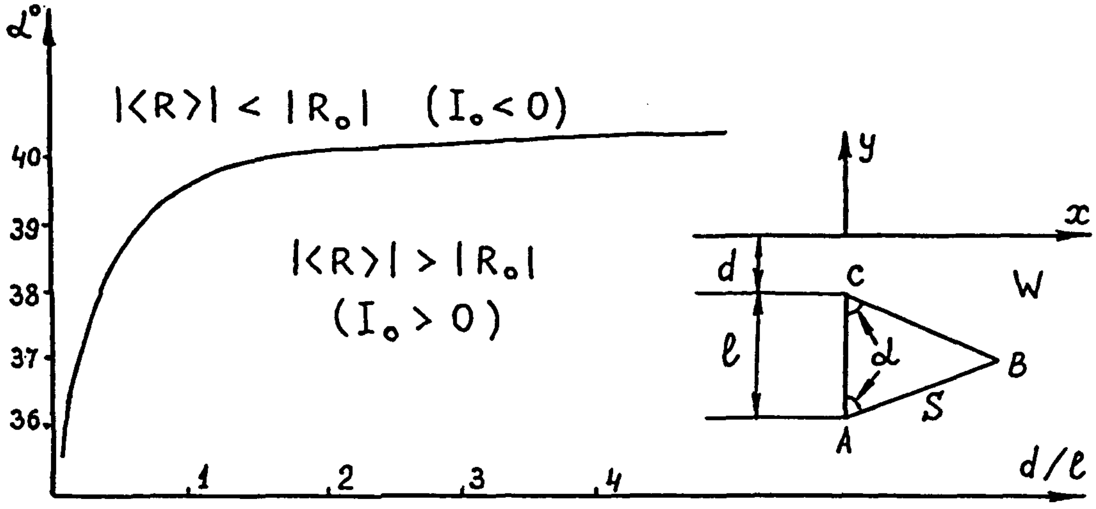

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$|\langle R\rangle| > |R_0|$
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Figure 1

VORTICAL FLOW PAST SLENDER BODIES AT INCIDENCE: EXISTENCE, UNIQUENESS, BIFURCATION AND STABILITY

by

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Vortical flows past slender bodies at high angles of attack play an important role in aerodynamics. The vortical flow has a marked effect on the forces and moments acting on an aerodynamic vehicle and can often determine the ultimate stability and performance of the vehicle. The study of vortical flows has thus fascinated aerodynamicists for many years, following early analytical studies of the flow past slender wings in the early 1950's. However, a particular problem arises with slender bodies in that the flow can spontaneously develop an asymmetric separation, even though the body is laterally symmetric and being flown without yaw. There has been considerable debate over the origin of this asymmetry with surface imperfections, asymmetric boundary layers, hydrodynamic instabilities and a range of other mechanisms being suggested as the root cause of the asymmetry. It is only recently that theoretical methods have been applied successfully to this problem of asymmetry and the underlying mechanism of asymmetry finally identified.

The paper will present the main results for the properties of the vortical flows past slender bodies as predicted by semi-analytical methods in the framework of slender-body theory. The paper will concentrate on inviscid results, where the separation is represented by the vortex-sheet or isolated vortex model and the position of separation from the body surface is prescribed. The relationship between these solutions and fully coupled viscous solutions for laminar and turbulent flows and the role of asymptotic models of boundary-layer separation will be discussed.

The main mathematical problem in computing the flow is finding the strength and location of the vortex sheet or isolated vortex core representing the separated flow. The vortex system must be force free by aligning itself with the local flow direction. The problem is essentially a nonlinear free boundary value problem, and a particularly accurate solution procedure will be described. This is based on a multidimensional extension of the Newton-Raphson iterative scheme. An important part of this iterative method is the construction of a Jacobian matrix of partial derivatives that reflect the rate of change of error (in terms of force on the vortex system) in the solution with changes in position of the vortex system.

A major finding of this study is the prediction of the development of asymmetric solutions for the vortical flow past symmetric bodies via a process of bifurcation from the symmetric solutions. Nonunique solutions are found for certain combinations of body shape and incidence. The bifurcation of the solutions manifests itself as a singular Jacobian matrix in the Newton iterative scheme, and this is used to track the onset of asymmetry in the symmetric solutions.

To determine which of the multiple solutions is likely to occur in a real flow, a stability analysis of some of the solutions have been performed. It will be shown that the eigenvalues of the Jacobian matrix of the Newton scheme determine the stability of the solutions. It will also be shown that an exchange of stabilities takes place with the appearance of the asymmetric solutions. As a consequence of this the asymmetric flows, where they exist, are found to occur naturally in preference to the symmetric flow. The onset of asymmetric flows is thus predicted to be an essentially inviscid phenomenon, and does not require viscous effects or surface imperfections to explain it. Results will also be given for the effect of body shape on the development of the asymmetric flows, and it will be demonstrated that body shape has a powerful effect on the suppression of the asymmetry - a useful result for the practicing aerodynamicist.

As well as non-unique solutions, the breakdown of steady, conical flow solutions of the flows past conical bodies has been found - i.e., non-existence of solutions under certain circumstances. The role of this nonexistence of steady solutions as a precursor to vortex shedding from the body will be discussed.

Finally, some recent results from various CFD calculations of separated flows past slender bodies will be described, to illustrate the important role that the analytical studies of vortical flows have played in identifying realistic solutions from the flow codes, and the role of analytical techniques in determining global properties of the vortical flows, rather than the 'spot' solutions given by CFD.

WEAKENING OF THE CUMULATIVE PHENOMENON AND SHOCKS IN TRANSONIC FLOWS

by

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The equations of inviscid gas motion are known to possess solutions that describe steady, shock-free, transonic flows in the following situations:

1) through an internal compression intake,

2) through an external compression intake,

3) with a local supersonic region,

4) with a local subsonic region,

5) with a closed sonic line (non-isentropic flow).

In practice some of the above flows appear to be unstable, so that any deviation of wall shape from a shock-free configuration produces a shock wave in the flow, although possibly a very weak one. Transonic flow stability with respect to deviations from the steady-wall-shape deviations has been treated mathematically in a number of papers; however, the causes of the flow instability and of the shock were not made quite clear.

Consider a smooth, plane, steady, transonic flow described by the velocity potential $\Phi(x, y)$. A disturbance $u(x, y)$ of $\Phi(x, y)$ satisfies the following linear equation derived from the full-potential one:

$$\frac{\partial}{\partial\Phi}\left[S(\Phi,\Psi)\frac{\partial u}{\partial\Phi}\right]+\frac{\partial}{\partial\Psi}\left[T(\Phi,\Psi)\frac{\partial u}{\partial\Psi}\right]=0. \tag{1}$$

Here $\Psi(x, y)$ is the steam function; S and T are known functions defined by the given flow: $T > 0$ in the flow field, $S > 0$ in the subsonic region, and $S < 0$ in the supersonic region. Thus, equation (1) represents a mixed-type problem.

In Kuz'min (1992a), a *mathematical study of boundary-value problems* for equation (1) was carried out using a priori estimates in Sobolev spaces. Let A_i be the points of the sonic line where the flow decelerates and two situations may be distinguished: either

(i) the velocity vector is orthogonal to the sonic line (points A_{i2}), or

(ii) the A_i are endpoints lying on a solid wall, and here the angle δ of the sonic line emanation from the wall is greater than $\pi/2$ with respect to the flow direction (points A_{i1}).

As shown in Kuz'min (1992a), there exist singularities of the solutions to the boundary-value problem at the points A_i. This result establishes the following linear stability conditions for transonic flows:

(i) the orthogonality points A_{i2} are to be absent in the flow, and

(ii) a proper wall perforation is to be implemented near the endpoints A_{i1} to prevent singularities at these points.

The latter singularities are caused virtually by a cumulative phenomenon, that is associated with the increase of disturbance amplitudes in the narrowing portion of the supersonic region.

Thus, flows 1 and 5 (indicated above) are unstable because of the presence of the orthogonality points A_{i2}. Flow 4 appears to be linearly stable. As for flows 2 and 3, one can employ a proper wall perforation and control to provide stability. Hereafter, we describe some results in detail just for flows 2 and 3.

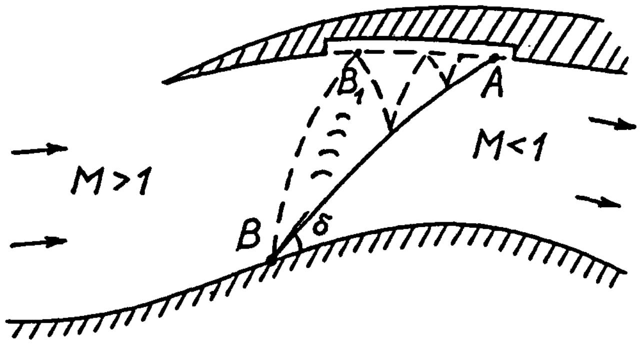

Figure 1

Consider a ***smooth, decelerating, transonic flow*** through a curvilinear channel (see Figure 1). The gas velocity is supersonic at the inlet and subsonic at the outlet. We prescribe an initial disturbance at the inlet and aim at finding the disturbance field in the channel. According to linear theory (Kuz'min, 1992a), a weak singularity (namely, a discontinuity in first-order velocity derivatives) arises at the upstream endpoint b of the sonic line BA; that singularity propagates along the dashed characteristics shown in Figure 1. Computations, moreover, reveal either a compression or rarefaction wave generated on the sonic line near point B. The amplitude of this wave, the so-called δ-wave, depends on the angle δ of the sonic line inclination to the wall. If $\delta = \pi/2$, then the δ-wave yields a velocity discontinuity on the bow characteristic BB_1, and we obtain a linearized version of the shock wave. It is exactly such a situation that occurs at the center of the internal compression intake.

Computations confirm the existence of the cumulative phenomenon in the supersonic region. When an active control is implemented on a portion of the upper wall confining the point A (see Figure 1), and Darcy's law is employed there as a boundary condition, computations show decaying oscillations of the velocity disturbance; thus, we obtain a continuous disturbance field. This illustrates a prevention of the cumulative phenomenon by the control, which offsets the disturbance energy.

In addition, the velocity disturbance is shown to admit the following asymptotic representation in the vicinity of the endpoint B of the sonic line (Kuz'min, 1992b):

$$\lambda^1_{sonic\ line} = (\tan\delta)^{1-\gamma/3}\, \psi^{-1+2\gamma/3}\, f_o R(\gamma,\lambda_\Phi) + \ldots \quad \text{if} \quad u|_{BB_1} = f_o \Psi^\gamma + o(\Psi^\gamma). \tag{2}$$

Here λ^1 is the disturbance of the nondimensional velocity $\lambda = v/a_*$ and R is a bounded function.

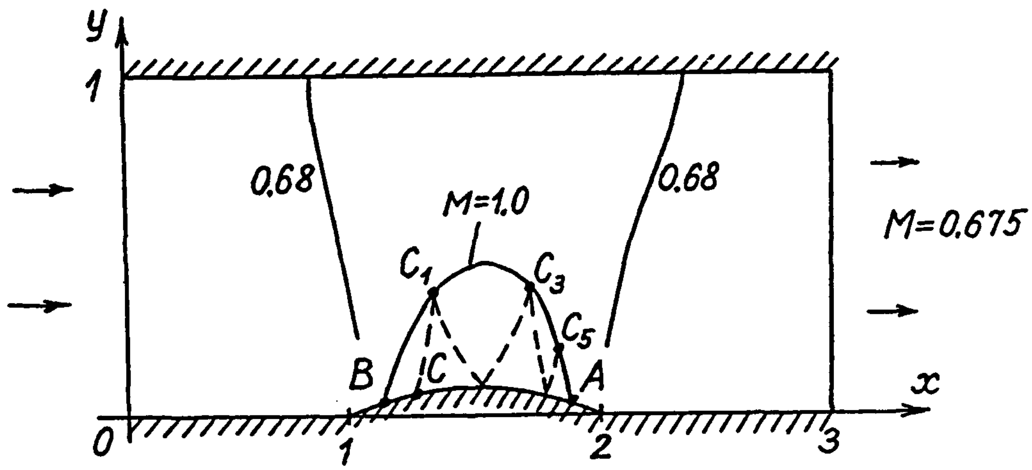

Figure 2

We now turn to ***transonic flow through a channel with a local supersonic region.*** The aim is to study the details of the shock rise in the supersonic region; we deal here with the nonlinear full-potential equation. First, a shockless bump utilizing Sobieczky's fictitious gas method (Sobieczky and Seebass, 1984) was designed. The starting channel geometry with a circular bump, of chord length 1 and height 0.1, was the same as considered by Ni (1981). The shock-free flow field was computed with the method of characteristics is depicted in Figure 2.

Let us now add the disturbance

$$\Delta y = Y\left[1 - \cos 2\pi\frac{x-a}{b-a}\right] \tag{3}$$

to the shockless wall shape in an interval $a < x < b$ containing the point C (see Figure 2). If Y is sufficiently small, then the shock wave origin is located in an arbitrarily small neighborhood of the point A. This confirms the conclusion of linear theory that disturbance cumulation results in a shock close to A. If the amplitude Y of the wall disturbance increases to a certain level, the origin of the shock wave jumps to the point C_3 due to the combined effects of the cumulative phenomenon and nonlinear instability. In all cases the shock origin is located near the sonic line; it is accounted for by nonlinear instability, which develops more rapidly near the portion of the sonic line (where the flow decelerates) than in the remaining part of the supersonic region.

In addition, the shockless bump with the added disturbance (3) was treated under an active control implemented in $1.48 \leq x \leq 1.78$. The adjustment of the active control was carried out by supporting the same pressure distribution in the cavity beneath the perforation as the pressure distribution on the undisturbed shockless bump. Such a control turns out to eliminate the shock wave caused by the disturbance (3). For the case of the circular bump (Ni, 1981), using the aforementioned active control, we again obtain a transonic flow which is nearly shock-free.

In recent studies of perforated airfoils aimed at weakening the shock (Chung-Lung et al., 1989), the perforation was placed traditionally in the shock root region of the airfoil's surface. However, our studies show it expedient to implement a proper control along the entire narrowing portion of the supersonic region. In a number of papers devoted to shock-free airfoil design, the target flow pressure/velocity component used was prescribed throughout the surface of the airfoil (Giles and Drela, 1987). Such a prescription is incorrect from a mathematical viewpoint and must produce internal shocks, because it does not absorb perturbations and, hence, does not prevent the cumulative phenomenon, even though the pressure varies smoothly on the surface of the airfoil.

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STRUCTURE OF THE THREE-DIMENSIONAL TURBULENT BOUNDARY LAYER

by

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The asymptotic structure of a three-dimensional turbulent boundary layer in the limit of large Reynolds number is considered. As in two-dimensional flows, the boundary layer is divided into two layers, viz., the inner wall layer where turbulent and viscous shear stresses are comparable in magnitude, and a relatively thicker outer layer where viscous stresses are negligible. The three-dimensional boundary layer may be best described in a streamline coordinate system as shown in Figure 1. At the edge of the boundary layer, the velocity U_e is aligned with the streamwise coordinate x_1. The cross-stream and normal coordinates, x_2 and x_3, respectively, complete the orthogonal system. In general, the external streamline is curved which sets up a cross-stream pressure gradient. Under its influence, a cross-stream velocity component, u_2 and x_3, respectively, complete the orthogonal system. In general, the external streamline is curved which sets up a cross-stream pressure gradient. Under its influence, a cross-stream velocity component, u_2, develops within the boundary layer and, consequently, the velocity vector rotates away from its direction at the boundary-layer edge. Both components of velocity, u_1 and u_2, must satisfy the no-slip condition at the wall; hence, the cross-stream velocity attains its maximum within the boundary layer. The angle θ is denoted as the velocity skew angle, and its value at the wall, θ_w, is evaluated using the L'Hopitals rule and is called the wall skew angle.

An important result from the asymptotic analysis is that the wall skew angle θ_w scales on the friction velocity u_τ which is defined as $u_\tau = \sqrt{\tau_w/\rho}$ where τ_w and ρ are the wall shear stress and density, respectively. The streamwise velocity distribution is similar to that in two-dimensional boundary layers. Specifically, in the wall layer

$$u_1 = u_* U^+ + \ldots, \tag{1}$$

where $u_* = u_\tau \cos\theta_w / U_e$ and

$$U^+ = 0 \quad \text{at} \quad y^+ = 0; \quad U^+ \sim \tfrac{1}{\kappa} \log y^+ + C_i \quad \text{as} \quad y^+ \to \infty. \tag{2}$$

Here y^+ is the usual scaled normal variable, and κ and C_i are assumed to be 0.41 and 5.0, respectively. In the outer layer, the expansion for u_1 takes on the usual defect form, viz.

$$u_1 = U_e + u_* \frac{\partial F_1}{\partial \eta} + \ldots, \tag{3}$$

where

$$\frac{\partial F_1}{\partial \eta} \sim \tfrac{1}{\kappa} \log \eta + C_o \quad \text{as} \quad \eta \to 0, \tag{4}$$

to match the velocity in the wall layer. Here η is the normal coordinate scaled with respect to Δ_o which represents the boundary-layer thickness. Matching the velocity in

(1) and (3) in the overlap region, and using (2) and (4), gives the match condition,

$$\frac{1}{u_*} = \frac{U_e}{u_\tau \cos\theta_w} = \frac{1}{\kappa} \log\,(\mathrm{Re}\; u_\tau \Delta_o) + C_i - C_o. \tag{5}$$

It may be noted from (1)–(5) that the structure of the streamwise velocity profile is similar to that in two-dimensional turbulent boundary layers.

It emerges from the asymptotic analysis that an expansion to two orders is necessary in order to describe the cross-stream velocity u_2. Specifically, in the wall layer,

$$u_2 = u_*^2 \theta_* U^+ + \ldots \tag{6}$$

where θ_* is the scaled wall skew angle defined by

$$\theta_* = \frac{\tan\theta_w}{u_*} \sim O(1). \tag{7}$$

In the outer layer,

$$u_2 = u_* \theta_* \left\{ G_1 + u_* G_2 + \ldots \right\}, \tag{8}$$

where

$$G_1 \sim 1,\; G_2 \sim \frac{1}{\kappa} \log \eta + C_o \quad \text{as} \quad \eta \to 0, \tag{9a}$$

$$G_1,\quad G_2 \to 0 \quad \text{as} \quad \eta \to \infty. \tag{9b}$$

It may be easily confirmed, using (5), that the outer- and wall-layer expansions match in the overlap region.

It is of interest to discuss the manner in which the cross-stream velocity attains its characteristic shape in Figure 1. In the outer layer close to the boundary-layer edge, the cross-stream velocity u_2 is dominated by the contribution from the leading-order term, G_1, and continuously increases in magnitude with decreasing distance from the wall. As the overlap region between the outer and the wall layers is reached, the second-order term, G_2, begins to make an increasingly negative contribution until the cross-stream velocity reaches its maximum value. A further decrease in distance from the wall reduces the sum of the $O(u_*)$ and $O\,(u_*^2)$ terms to $O(u_*^2)$ in the wall layer in much the same fashion as the sum of the $O(1)$ and $O(u_*)$ terms reduces the streamwise velocity in the outer layer to $O(u_*)$ in the wall layer.

Similarity equations for the outer layer are derived, and it emerges that a two-parameter family of similarity solutions exist, and these are the pressure-gradient parameters defined as

$$\beta_s = \frac{\delta^*}{u_\tau^2 \cos\theta_w} \frac{U_e}{h_1} \frac{\partial U_e}{\partial x_1},\quad \beta_c = \frac{\delta^*}{u_\tau^2 \cos\theta_w} K_2 u_e^2, \tag{10}$$

where δ^* is the displacement thickness, h_1 is the streamwise metric and K_2 is the curvature of the external streamline. Computed solutions of the outer layer equations, using a simple algebraic model consistent with the asymptotic results, are matched to an analytical wall-layer profile to produce composite profiles spanning the entire thickness of the boundary layer.

The influence of Reynolds number and cross-stream pressure gradient are investigated. It is found that the extent of collateral flow from the wall increases with increasing Reynolds number. In terms of the wall layer variables y^+, the location of the maximum cross-stream velocity from the wall increases with Reynolds number, but in terms of the outer-layer variable η, it decreases. This confirms that the location of the maximum cross-stream velocity from the wall is within the overlap region between the outer and inner wall layers.

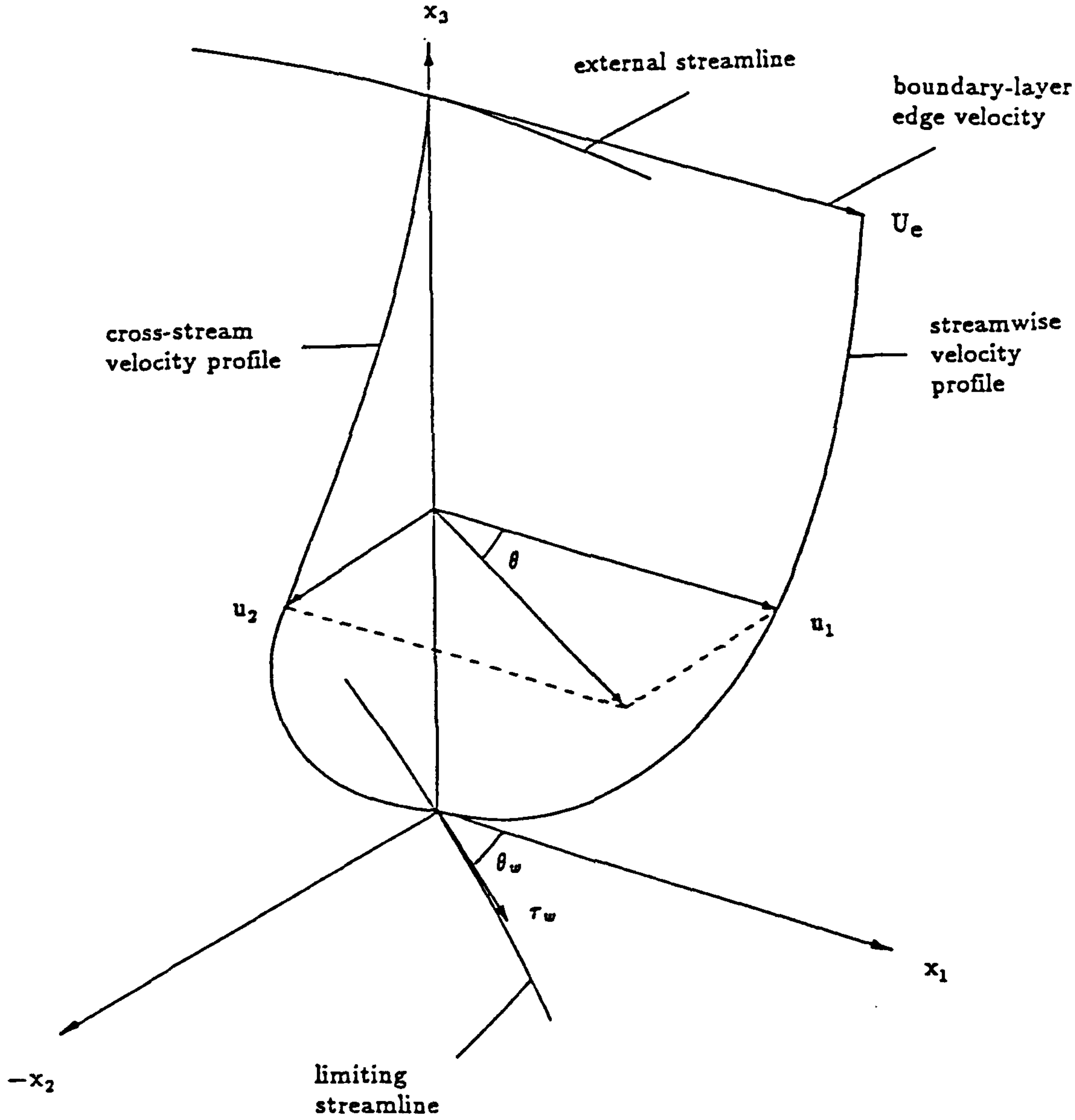

Figure 1. Schematic of a three-dimensional turbulent boundary-layer velocity profile.

ON THE LANDAU-GOLDSTEIN SINGULARITY AND MARGINAL SEPARATION

by

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A comprehensive analysis of a classic two-dimensional boundary layer is developed with the aim of revealing possible types of singularities related to separation. According to the basic assumption, the limit flow regime with the singular point of vanishing skin friction obeys an analytic solution where the frictional intensity approaches zero according to quadratic law rather than varying linearly with distance. Small deviations from the limit regime are described in terms of eigen-functions, three of which involve singularities. The flow field for all supercritical regimes is continued into a small region centered about the singular characteristic arising from the point of vanishing skin friction in the undisturbed limit solution. The solvability condition for a key boundary value problem posed in this region as a result of matching with the global-scaled solution upstream gives three different types of singularities of the Prandtl equation. According to the weakest type singularity, the skin friction varies as the square root of the cube of the local distance. The next type includes a sudden change in the wall shear stress derivative with respect to coordinate along the solid surface; it was ruled out of the basic limit solution. Then the famous Landau-Goldstein singularity with the skin friction being proportional to the square root of the local distance evolves. A still more complicated flow pattern may be composed of the singularity with the sudden change in the wall shear stress derivative superseded by the Landau-Goldstein singularity at some small distance downstream.

The flow patterns of such a kind have been clearly observed experimentally in wind-tunnel tests, where a local minimum in the wall shear stress preceded massive separation. Thus, the weaker singularity may be considered to be a prerequisite for incipient separation, giving rise to a short bubble confined by flow reattachment to the body surface; the Landau-Goldstein singularity marks reseparation of the reattached boundary layer. The widespread explanation of reattachment was associated with the onset of turbulence in the separated flow inside the short bubble. However, the same phenomenon can be predicted theoretically to occur in a laminar boundary layer suffering from turbulent pulsations.

An analysis of an interactive boundary layer within the framework of the triple-deck theory is also presented with an emphasis on the formation of separation zones.

CONSTRUCTION OF OPTIMAL CRITICAL AIRFOILS

by

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Gilbarg and Shiffman characterized optimum non-lifting critical airfoils, those which provide the largest freestream Mach number for a given thickness (or area), as having vertical segments at nose and tail connected by a sonic arc. We have calculated these shapes for a variety of Mach numbers using a hodograph formulation. An analytical solution to the problem is provided under the assumptions of transonic small disturbance theory (TSDT). For TSDT the vertical segments at nose and tail are replaced by $\frac{2}{5}$ power bodies. There is good agreement between exact calculations and (TSDT) shapes. A critical similarity parameter K_c is found. The work is extended with the constraint of a finite tail angle.

An accurate code based on Sells method is developed to calculate the flow past given shapes so that a comparison can be made with conventional airfoils such as NACA00XX. The results show an improvement of 2-5% in critical Mach number.

Next the considerations are extended to the lifting case. The topology of the hodograph is shown under (TSDT). Calculations are carried out based on the idea that long sonic arcs from the nose are desirable. In this way a series of lifting airfoils has been found whose critical Mach number is close to that of the non-lifting airfoils.

VISCOUS-INVISCID LAMINAR INTERACTION NEAR THE TRAILING TIP OF AN AXISYMMETRIC BODY

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This paper discusses the flow field near the trailing-tip region of a slender body of revolution of finite length placed in a uniform incompressible stream when the characteristic Reynolds number of the flow is large. Under these conditions the well-known Mangler transformation is not applicable since the radius of the body can now be of the same size as the boundary-layer thickness. In this case, one must consider the effect of transverse curvature in the boundary-layer development, even though it is still permissible to neglect longitudinal curvature effects to leading order. In addition the flow in the immediate vicinity of the trailing tip needs special attention since now the body radius itself shrinks to zero.

Bodonyi, Smith and Kluwick (*Proc. Roy. Soc. Lond.* A 400, 1985) studies the classical boundary-layer development over a family of slender bodies of revolution of finite length with special emphasis on establishing the flow properties near the trailing tip of the body. They found that, in contrast to the case of two-dimensional flow, the axisymmetric boundary layer does not simply set up an arbitrary velocity profile at the onset of the trailing tip. Instead it develops an interesting multi-layer structure depending explicitly on the trailing-tip shape which they took to be of the form $(1-x)^n$ where $x=1$ corresponds to the trailing tip and $n>0$ is an index of the body shape. Different multi-flow structures were deduced depending on the value of the parameter n. They also gave an argument showing that the classical boundary-layer strategy breaks down near the trailing-tip as $x \to 1$ – with the streamwise interaction length scale being a function of both the Reynolds number and the parameter n, such that $1-x = O\,(Re^{-1} \ln Re)^{1/(4-6n)}$.

In this paper the viscous-inviscid pressure interaction structure is studied in detail for the case when $1/4 < n < 1/3$. In this range the pressure interaction first affects the viscous wall layer in such a way that the body radius and the wall layer thickness are comparable in size so that the interactive flow is no longer quasi-planar. Here the governing equations are the axisymmetric boundary-layer equations subject to appropriate boundary conditions for the interaction. It should be noted that the criterion on n restricts only the incoming form of the surface shape, at the upstream end of the interaction, not the shape throughout the interaction region. In particular, we can study body shapes with blunt or abruptly cut-off trailing tips, i.e., base flow problems, directly within the framework of the interaction problem.

Specifically, we shall present the interaction flow structure for the trailing-tip region for body shapes with n ranging between 1/4 and 1/3. Furthermore, numerical and analytical solutions of the appropriate governing equations will be presented for slender axisymmetric bodies with different types of trailing-tip shapes.

THE GENERATION OF TOLLMIEN-SCHLICHTING WAVES BY FREESTREAM TURBULENCE

by

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Laminar-turbulent transition is an extraordinarily complicated process consisting of a number of events. It starts with the transformation of external disturbances into internal oscillations of the boundary layer taking the form of well-known Tollmien-Schlichting waves. In relatively quiet flow, their initial amplitudes are not high enough to provoke immediate transition.[1] Tollmien-Schlichting waves must first amplify in the boundary layer to bring into play the nonlinear effects characteristic of the transition process. Numerous experiments have clearly revealed that the extent of the amplification region is strongly dependent not only on the amplitude and/or the spectrum of external disturbances but also on their physical nature. Some of the disturbances easily penetrate into the boundary layer, others do not. To study these differences, the problem of the boundary-layer receptivity to external disturbances was formulated by Morkovin (1969) as a key problem in the laminar-turbulent transition. The main objective of the investigations in that field is the determination of amplitudes of generated Tollmien-Schlichting waves, and as a result the elucidation of those types of external disturbances which easily turn into Tollmien-Schlichting waves.

From the mathematical point of view, the receptivity problem appears to be much more difficult as compared with the stability problem. The latter is associated with the solution of the Orr-Sommerfeld equation, while the former calls for solution of a boundary value problem deriving from the Navier-Stokes equations. To date direct numerical simulations of the full Navier-Stokes equations appears to be very difficult as far as unstable boundary-layer flow at high Reynolds numbers is concerned. On the other hand, asymptotic methods are well-suited for this type of analysis.

The first paper on the topic was published by Terent'ev (1981). His analysis was devoted to Tollmien-Schlichting wave generation by a vibrator installed on the body surface. To describe the process, he used an unsteady version of the triple-deck theory, which was known to describe the asymptotic structure of Tollmien-Schlichting waves. As a result, an explicit formula was obtained for the amplitude of Tollmien-Schlichting waves propagating in the boundary layer downstream of the vibrator.

The problem of the Tollmien-Schlichting wave generation by sound has been considered by Ruban (1984) and independently by Goldstein (1985). Effective transformation of external disturbances into Tollmien-Schlichting waves is possible if resonance conditions are satisfied. For boundary-layer flow, the resonance supposes coincidence not only of frequencies, but also of the wave number of external and internal disturbances. This coincidence may be easily achieved in the problem considered by Terent'ev (1981), since the frequency of the vibrating part of the surface and its extent are independent of each other. If an acoustic wave plays the role of external disturbance, it is possible to choose its frequency in a proper way, but the wave length will be much greater than that of the Tollmien-Schlichting wave. In order to introduce the necessary length scale into the problem, Ruban (1984) and Goldstein

[1] This is just the case, for example, for aircraft flying in atmosphere.

(1985) supposed that the body surface is not absolutely smooth, and the acoustic wave is interacts with disturbance introduced into the boundary layer by wall roughness. As a result of their analysis, they concluded that effective generation of Tollmien-Schlichting waves takes place if the length of the roughness is of the order of $Re^{-3/8}$, where Re is Reynolds number.

The present work is devoted to the third possible way for Tollmien-Schlichting wave generation, namely that associated with transformation of free-stream turbulence into boundary-layer flow oscillations. For simplicity uniform turbulence interacting with flat plate, boundary layer will be considered. The problem was first analyzed by Hunt and Graham (1978). They considered the inviscid process of the velocity field deformation near the flat plate on account of slip condition. Under the assumption that the vorticity field is not influenced by the wall and is just the same as in free stream, it has been shown that the vortex boundary layer forms near the flat plate. Its thickness is of the order of $Re^{-1/4}$. Near the bottom of the layer, velocity fluctuations have maximum amplitude, but the pressure gradient remains zero. Investigation of the flow inside viscous boundary layer with thickness $O(Re^{-1/2})$ under these circumstances was carried out by Guliaev et al. (1989). As a result, they found out that disturbances do not penetrate into the boundary layer and so do not generate Tollmien-Schlichting waves.

We intend to show that this assertion is not correct. The mistake follows from the assumption (see Guliaev et al., 1989) that the vorticity field inside the vortex boundary layer is undisturbed and coincides with that upstream of the flat plate. More careful analysis based on the application of matched asymptotic expansions technique for the solution of the Navier-Stokes equations will be used to solve the problem. As a result it will be shown that turbulence deformation takes place in the vortex boundary layer leading to nonzero pressure gradient fluctuations near the outer edge of the viscous boundary layer. Explicit formula will be obtained for the amplitudes of generated Tollmien-Schlichting waves for different types of surface roughness.

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NONLINEAR MODULATION OF INSTABILITY MODES IN SHEAR FLOWS

by

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We report results of two different, but related, problems: (a) the nonlinear evolution of a pair of initially linear oblique waves in a high Reynolds number shear layer, and (b) the nonlinear development of a wave packet in an unstable boundary layer. In both cases fixed frequency disturbances are assumed to evolve downstream in a linear fashion until in the neighborhood of the upper branch nonlinear effects can no longer be neglected.

For the first problem, attention is focused on times when disturbances of amplitude ϵ have $O(\epsilon^{2/3}R)$ growth rates, where R is the Reynolds number based on shear layer width. The development of a pair of oblique waves is then controlled by non-equilibrium critical-layer effects (Goldstein and Choi, 1989). Viscous effects are included by studying the distinguished scaling $\epsilon = O(R^{-1})$. An amplitude equation of integro-differential form is derived. When viscosity is not too large, solutions to the amplitude equation develop a finite-time singularity, indicating that an explosive growth can be induced by nonlinear effects. Increasing the importance of viscosity generally delays the occurrence of the finite-time singularity and sufficiently large viscosity may lead to the disturbance decaying exponentially. By studying the very viscous limit, a link between the unsteady critical-layer approach to high-Reynolds-number flow instability, and the wave/vortex approach of Hall and Smith (1991), is identified. In this limit, the amplitude equation reduces to a simpler form, but still contains a nonlocal nonlinear term. Further, the critical layer splits into two regions: a steady viscous critical layer, and a diffusion layer accommodating the vortex flow. This structure turns out to be related to the second problem studied in this paper.

In the second problem, a wave packet of almost two-dimensional Tollmien-Schlichting waves is studied. The amplitude of the wave packet is a slowly varying function of spanwise position, as well as of the streamwise spatial variable. Attention is focused on the upper-branch-scaling regime. We show that dominant nonlinear effects come from both the critical-layer and the diffusion layer - in the latter layer nonlinear interactions influence the resolution by producing a spanwise-dependent mean-flow distortion. The evolution is governed by an integro-partial-differential equation containing a history-dependent non-linear term. A feature of the amplitude equation is that the highest derivative with respect to spanwise position appears in the nonlinear term. It is hoped that this study may offer an explanation for the wave packet splitting observed by Gaster and Grant (1975).

RECEPTIVITY OF A SUPERSONIC BOUNDARY LAYER TO SOUND NEAR THE LEADING EDGE OF A FLAT PLATE

by

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An asymptotic theory of subsonic boundary-layer receptivity near the leading edge was proposed by Goldstein (1983) and advanced by Kerschen et al. (1989). However, their results cannot be applied immediately to supersonic boundary layers since in this case the spectrum of normal modes has a new characteristic. In contrast with the subsonic case, the oscillation damping rates are small in the region lying upstream from the neutral curve (Fedorov, 1982). As the leading edge is approached, the normal modes of the boundary layer become synchronized with sound waves (Gaponov, 1985), and thus their frequencies and wave numbers are close to those of sound. For these reasons, the neighborhood of the leading edge appears to play the dominant role in excitation of instability at supersonic speeds. This assumption is confirmed by experiment (Maslov and Semenov, 1986) and also finds support in the strong influence of leading-edge bluntness on laminar-turbulent transition (Gaponov and Maslov, 1980).

We examine the excitation of instability near a semi-infinite flat plate situated in a uniform supersonic flow of velocity U_∞ and kinematic viscosity ν. We suppose that the flow is disturbed by a small-amplitude acoustic wave of frequency ω. At low values of the frequency parameter $F=\nu\omega/U_\infty^2$, the disturbance field has a structure schematically shown in Figure 1. Cartesian coordinates x, y, z (referred to U_∞/ω) can be resolved into the following scales variables:

$$x=(x_1, \epsilon^{-2}x_2, \epsilon^{-4}x_3), \quad y=(\epsilon y_1, y_3, \epsilon^{-1}y_2).$$

Here $\epsilon=F^{1/4}\Delta^{1/2}$ is a small parameter. The parameter $\Delta=\delta^*(x\nu/\omega)^{-1/2}$ is introduced into the scaling to take into account the strong dependence of the boundary-layer displacement thickness δ^* on the Mach number M and wall-temperature factor T_f. The "rapid" scale $x_1=O(1)$ corresponds to the acoustic wavelength; the maximum of instability is located in the downstream region $x_3=O(1)$, and the diffraction zone $x_2=O(1)$ is responsible for the transformation of external disturbances into the boundary-layer normal modes.

At first we consider the process of receptivity to an acoustic wave propagating downstream parallel to the plate surface. The solution within the diffraction zone can be represented as $p_2(x_2, y_2)\cdot e^{iS}$, where $S=\alpha_{1,2}x+\beta z-\omega t$ is the phase of the external wave with longitudinal wavenumbers $\alpha_{1,2}=[M^2\pm(M^2+\beta^2(M^2-1))^{1/2}]/(M^2-1)$ corresponding to forward and backward wave fronts. Applying the method of matched asymptotic expansions in the y-direction, we can formulate the problem for the pressure amplitude p_2; it is reduced to an integral equation for the pressure amplitude

$p_1(x_2) = p_2(x_2, 0)$ within the boundary layer under the diffraction zone given by,

$$p_1(x_2) - \lambda \int_0^{x_2} \sqrt{\frac{\xi}{x_2 - \xi}}\, p_1(\xi) d\xi = 1. \tag{1}$$

The parameter λ depends on the boundary-layer velocity $U(\eta)$ and temperature $T(\eta)$ distributions with respect to the self-similar variable $\eta = y_1/\sqrt{x_2}$

$$\lambda = \frac{-(\alpha_{1,2}-1)^2}{[2\pi i(M^{-2}(\alpha_{1,2}-1) - \alpha_{1,2}]^{1/2}} \int_0^{\infty} \left[\frac{T\cdot(\alpha_{1,2}^2+\beta^2)}{(\alpha_{1,2}U-1)^2} - M^2\right] d\eta.$$

The solution of equation (1) is represented in the series form

$$p_1 = \sum_{n=0}^{\infty} a_n \pi^{n/2}(\lambda x_2)^n, \quad a_0 = 1, \quad a_n = \prod_{j=0}^{n} \frac{\Gamma(j+1/2)}{\Gamma(j+1)}. \tag{2}$$

The asymptotic structure of the solution at large x_2 can be found by replacing the power series by an integral in the complex n-plane. If $\mathrm{Re}(\lambda) > 0$

$$\begin{aligned} p_1 &= K_0(\pi\lambda^2 x_2^2)^{1/8} \exp(\pi\lambda^2 x_2^2/2) - (\pi\lambda x_2)^{-1} + \ldots, \\ K_0 &= (8\pi)^{1/4}A, \quad A = \lim_{n\to\infty}[n^{-1/8} a_n \sqrt{\Gamma(n+3/2)}] = 0.935\ldots, \end{aligned} \tag{3}$$

If $\mathrm{Re}(\lambda) < 0$, the asymptotic solution contains only the second term of (3), corresponding to the acoustic field near the wall zone. The first term is the "seed" for the normal modes of the boundary layer. The comparison between the rigorous solution (2) and its far-downstream asymptotic form (3) presented in Figure 2 shows that quite good agreement is observed at $|\lambda x_2| > 2$.

In order to extend the solution in the boundary layer with variables $(x_3, y_3) = 0(1)$, we employ the formalism developed by Nayfeh (1980) and Zhigulev and Tumin (1987). The perturbation vector consisting of the pressure p, temperature θ, velocity vector u, and their derivations with respect to y_3 can be represented in the form

$$\begin{aligned} z &= a(x_3)\cdot[z_0 + \epsilon^4 z_1 + \ldots)\cdot\exp(i\hat{S}), \\ \hat{S} &= \epsilon^{-4}\int_0^{x_3} \hat{\alpha}(x_3)dx_3 + \beta z - \omega t. \end{aligned} \tag{4}$$

The perturbation amplitude z_0 and the eigenvalue $\hat{\alpha}$ are determined from the boundary-value problem, while the behavior of the amplitude function $a(x_3)$ is obtained from the condition of solvability for the second-order approximation z_1. Analysis of the problem shows that there are two normal modes synchronizing with the acoustic waves at *the* leading edge. The asymptotic forms of eigenvalues and amplitude functions for such modes at small x_3 are: $\hat{\alpha} = \alpha_{1,2} + \pi\lambda^2 x_3 + \ldots$, $a = Cx_3^{1/4} + \ldots$. Substituting these relations into the representation (4), we can get an explicit expression for the pressure perturbations in the boundary layer as $x_3 \to 0$,

$$p = C \cdot x_3^{1/4} \exp(\lambda^2 \epsilon^{-4} x_3^2) \exp(iS). \tag{5}$$

The exponential part of the asymptotic solution (3) coincides with the asymptotic form (5) correct to a constant multiplier, i.e. the zone of diffraction and far-downstream boundary layer overlap (shaded zone in Figure 1). By matching we obtain

$$C = \epsilon^{-1/2} K_0 \cdot (\pi\lambda^2)^{1/8}. \tag{6}$$

Thus we have determined the relation between the eigensolution (4) describing the first and second boundary-layer modes and the parameters of the external acoustic wave propagating downstream parallel to the plate surface.

The proposed approach can be generalized to inclined acoustic waves. Variation of the external forcing leads to variation of the right-hand side of equation (1). We obtain the explicit form of the coupling coefficient C for acoustic wave with *any* value of longitudinal wave number α from the acoustic range: $\alpha \geq \alpha_1$, $\alpha \leq \alpha_2$. It coincides with the expression (6) in which the constant K_0 is replaced by a function

$$K_a = K_0 \left\{ \epsilon^{-1} \exp(\pm i\pi/4) \frac{i\pi^{1/4} \mu(\alpha)(\alpha_{1,2} - 1)}{\lambda^{1/2} A^2 \sqrt{\pi(M^2-1)(\alpha_1\alpha_2)(\alpha - 1)}} \psi(r) + \varphi(r) \right\},$$

$$r = -i(\alpha_{1,2} - \alpha)\epsilon^{-2}(\pi\lambda)^{-1}.$$

The functions ψ and φ are given by the following power series,

$$\varphi(r) = \sum_{n=0}^{\infty} \frac{\pi^{n/2} r^n}{a_n n!}, \quad \psi(r) = \sum_{n=0}^{\infty} \pi^{n/2} a_n r^n,$$

and the y-component of the wave vector is $\mu(\alpha) = [(M^2 - 1)(\alpha - \alpha_1)(\alpha - \alpha_2)]^{1/2}$.

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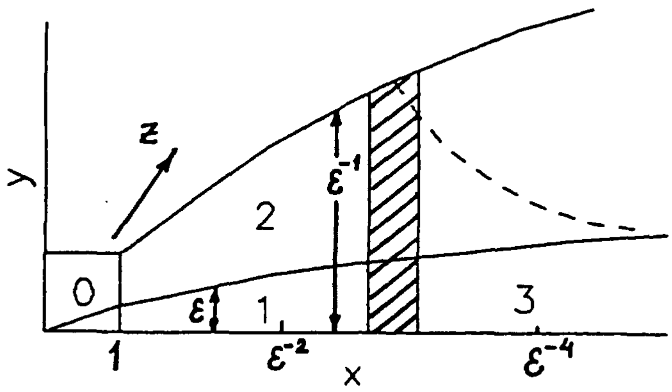

Figure 1. Symptotic structure of the disturbance field.

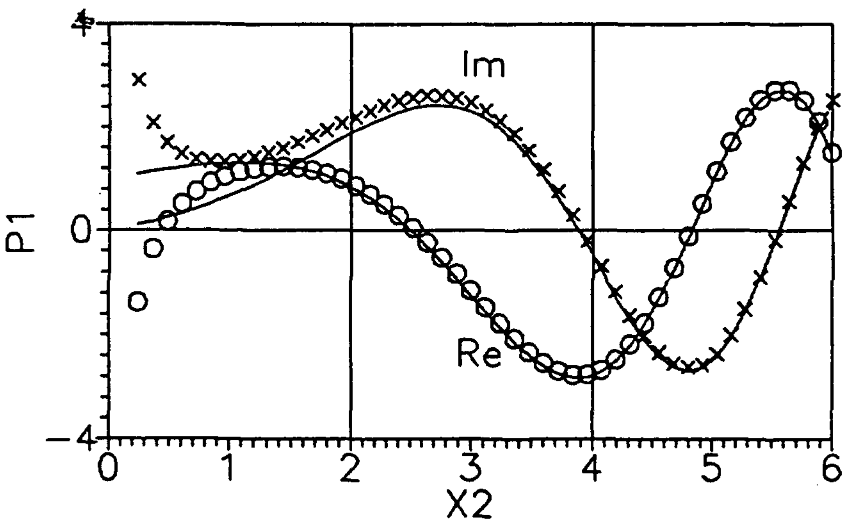

Figure 2. Comparison of rigorous (solid lines) and asymptotic (symbols) solutions of equation (1); $\lambda = 0.25 + i0.25$.

INFLUENCE OF THE ENTROPY LAYER ON VISCOUS TRIPLE DECK HYPERSONIC SCALES

by

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Inviscid-viscous interaction on a flat blunted plate in the weak hypersonic regime (i.e., when wall-perfect fluid pressure nondimensionalized by free stream pressure, say ω, is much greater than the classical hypersonic viscous interaction parameter χ_{∞}) is studied on the triple deck scales (Stewartson, 1974, and Neiland, 1990). We seek to delineate the influence of an asymptotically small nose bluntness (which creates a thin layer of perfect fluid called entropy layer: Guiraud, Vallée and Zolver (1965)) on the flow structure near a laminar separation.

We outline some cases depending on the relative sizes of the upper deck and the entropy layer.

1. THE ENTROPY LAYER IS SMALLER THAN THE UPPER DECK

In this case (Lagrée, 1991), a fourth deck must be introduced, lying between the main and the upper deck, which is the entropy layer itself (it is characterized by small density, say gauged by $r\rho_{\infty}{}^*$ where r is small, and by small thickness, gauged in Von Mises transverse variable by $d^*\rho_{\infty}{}^*U_{\infty}{}^*$; we noted $d = d^*/L^*$ the ratio of tip bluntness versus the longitudinal scale).

As a result, we obtain a new fundamental equation of the triple deck written in standard scales. The reduced lower deck equations are identical to those of classical theory:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial^2 u}{\partial x^2}. \tag{1}$$

As boundary conditions, we have the no slip velocity condition and,

$$u(x) \to y + A(x), \quad \text{as} \quad y \to \infty. \tag{2}$$

However, the pressure displacement relation is different and

$$p(x) + \eta\frac{dp(x)}{dx} = -\frac{dA(x)}{dx}. \tag{3}$$

The infinitesimally small parameter η (directly proportional to the nose blunting by the thickness of the entropy layer and inversely proportional to the upper deck's scale) gauges the departure from classical theory as given below; here Fp denotes the finite part of the integral which is performed through the entropy layer and

$$\eta = (d/\Psi)Fp\left\{\int_0^\infty \frac{1}{r\,\hat{p}\,(\hat{\psi})}\,d\,\hat{\psi}\right\}, \text{ and } O(\eta) = O(dr^{-1}\chi_\infty^{3/4}\,\omega^{-1/2}). \tag{4}$$

This may be compared with another entropy effect in Sokolov (1983) and to the wall temperature effect in Brown, Cheng & Lee (1990).

2. THICKER ENTROPY LAYER, TWO PARTICULAR CASES:

2.1 *Entropy Layer and Upper Deck are the Same*

When the upper deck is the entropy layer, the complete equations of perturbation have to be solved in the upper deck, the density is given by the density profile of the entropy layer (Guiraud et al., 1965):

$$\frac{\partial v}{\partial \xi} = -\frac{\partial p}{\partial \psi}, \; \frac{\partial v}{\partial \psi} = \frac{1}{\hat{p}}\frac{\partial p}{\partial \xi}. \tag{5}$$

The longitudinal gauge is imposed by the size of the upper deck.

2.2 *Entropy Layer is Thicker Than Upper Deck*

When the upper deck is smaller than the entropy layer, we find again the classical scale with but different scales (Lagrée, 1990) because the propagation takes place in a layer of very small constant density (r), for example, the longitudinal gauge is $(\omega = M_\infty{}^2 d^{2/3})$:

$$x_3 = \lambda^{-5/4}(\gamma-1)^{3/2}\, s_W^{3/2}(\chi_\infty/\omega)^{3/4} r^{3/8}. \tag{6}$$

3. NUMERICAL RESOLUTION

These problems have to be solved numerically, and to this end, we choose an iterative method based on standard inverse Keller Box method for Prandtl equations, plus numerical resolution of the perfect fluid by integration of (3) or by a MacCormack scheme (case of complete relations (5)), plus a revisited Le Balleur (1978) "semi-inverse" relaxation method. This permits strong coupling.

Results for η small (first case relation (3)) are close to those of Cheng et al. (1990). The linear solution predicts that the Lighthill eigenvalue k increases with η, so the curve slope steepens with η. The separation bubble size appears to increase with η. Results for second case (complete resolution of (5)) show firmly that increasing the entropy layer diminishes the bubble, and that decreasing it increases the bubble (confirming qualitatively the relation (3)).

4. CONCLUSION

To summarize, a rough sketch of small nose bluntness influence may be drawn. For $\eta \ll 1$, the study of section 1 (equation (3)) may apply; raising η increases separation (numerically). For η of order one, this study fails and complete calculation of inviscid perturbation (equation (5)) through a thick entropy layer has been performed. It seems that the good trends are qualitatively obtained for thin and thick entropy layer. For larger bluntness (but always with d very small), section 2.2 suggests new scales. So increasing η first promotes growth of separated region, reduces k and diminishes the apparent interaction region, a further increasing lowers the scale of the separated region. This is qualitatively comparable with the experimental data of Holden (1971). The incipient angle separation is correlated with:

$$M_\infty \alpha (M_\infty^3 d)/\xi_\infty^2 \propto \left(\chi_\infty / \left((M_\infty^3 d)^{3/2}\right)\right)^a (d)^b. \tag{7}$$

Holden (1971), through a combination of parameters and experiment, found $a = -7/5$ and $b = 0$. We propose, deduced from triple deck scales, $a = -3/2$ and $b = -1/6\gamma$; these coefficients reflect the locality of the interaction.

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ASYMPTOTIC SOLUTION OF AN AXISYMMETRIC IDEAL-FLOW PROBLEM FOR A CAVITY APEX REGION

by

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In this paper a potential ideal fluid-flow with a cavity is considered. It is supposed that the flow is axially symmetric in the cavity apex region.

Let us introduce the Cartesian system of coordinates x, y, z with the origin being placed in the cavity apex. An asymptotic solution of the Euler equations for the limiting processes $R=\sqrt{x^2+y^2+z^2}\to 0$ or $R\to\infty$ is studied. The velocity components are referenced by an absolute value of velocity that occurs on the cavity shape. The velocity potential and cavity shape equations are represented in spherical polar coordinates R, β, α ($x=R\cos\beta$, $y=R\sin\beta\sin\alpha$, $z=R\sin\beta\cos\alpha$), by

$$\varphi=R\cos\beta+\varphi_1(R,\beta),\qquad \phi\equiv R\sin\beta-f(R\cos\beta)=0.$$

The functions φ_1, ϕ satisfy the following equations,

$$\frac{\partial^2\varphi_1}{\partial R^2}+\frac{2}{R}\frac{\partial\varphi_1}{\partial R}+\frac{1}{R^2}\frac{\partial^2\varphi_1}{\partial\beta^2}+\frac{\operatorname{ctg}\beta}{R^2}\frac{\partial\varphi_1}{\partial\beta}=0,$$

$$\nabla\phi\cdot\nabla(R\cos\beta+\varphi_1)=0,\qquad |\nabla(R\cos\beta+\varphi_1)|=1 \text{ at } \phi=0, \tag{1}$$

and the condition that $\nabla\varphi_1$ does not have a singularity in the flow region considered (outside the cavity).

Let us suppose that, in the case $|\ln R|\to\infty$, the following asymptotic representation for the φ_1 is valid:

$$\varphi_1\sim R^n g(t)G(t,\beta,n) \tag{2}$$

where n is an eigenvalue, $t=\ln R$ and

$$G(t,\beta,n)=O(1),\quad 0<\beta\le\eta.$$

$$|\ln g|\to\infty,\quad \left|g^{(m+1)}(t)\right|\ll\left|g^{m}(t)\right|,\quad m=0,1\ldots. \tag{3}$$

From (1), (2), we can find that the function G is governed by the equation

$$\overset{**}{G} + \text{ctg}\,\beta\overset{*}{G} + \left[n(n+1) + (2n+1)\left(\frac{g'}{g} + \frac{G'}{G}\right) + \frac{g''}{g} + \frac{G''}{G} + 2\frac{g'}{g}\frac{G'}{G}\right]G = 0,$$

where

$$(\ )^* \equiv \frac{\partial}{\partial\beta}, \quad (\ )' \equiv \frac{\partial}{\partial t}.$$

The solution of this equation can be represented as the following asymptotic expansion,

$$G(t, \beta, n) = G_0(\beta, n) + \frac{g'}{g}\,G_1(\beta, n) + O\left[\left(\frac{g'}{g}\right)^2\right] + O\left(\left|\frac{g''}{g}\right|\right).$$

The function $G_{01} = G_0 + (g'/g)\,G_1$ satisfies the Legendre equation

$$\overset{**}{G}_{01} + \text{ctg}\,\beta\,\overset{*}{G}_{01} + N(N+1)G_{01} = 0, \quad N = n + g'/g.$$

The solution of this equation can be expressed in terms of the hypergeometric function F by,

$$G_{01} = C_1 F\left(-\frac{N}{2}, \frac{N+1}{2}, \frac{1}{2}, \cos^2\beta\right) + C_2\cos\beta\,F\left(\frac{1-N}{2}, \frac{N+2}{2}, \frac{3}{2}, \cos^2\beta\right),$$

where $C_1(t)$, $C_2(t)$ are free coefficients.

In the case $\cos^2\beta \to 1$, the function G_{01} has the following asymptotic expansion,

$$G_{01} \sim a_1\left\{\left[\psi\left(-\frac{N}{2}\right) + \psi\left(\frac{1+N}{2}\right)\right] + \ln\sin^2\beta + \ldots\right\} +$$

$$+ a_2\cos\beta\left\{\left[\psi\left(\frac{1-N}{2}\right) + \psi\left(\frac{2+N}{2}\right)\right] + \ln\sin^2\beta + \ldots\right\},$$

$$a_1 = -C_1\frac{\Gamma(1/2)}{\Gamma(-N/2),\,\Gamma(1/2(N+1))}, \quad a_2 = -C_2\frac{\Gamma(3/2)}{\Gamma(1/2(1-N)),\,\Gamma(1/2(N+2))}.$$

In this formula $\Gamma(z)$ is Euler's gamma function, and $\psi(z) = \Gamma'(z)/\Gamma(z)$ is the logarithmic derivative of the gamma function.

From the condition that the problem solution must not have any singularity in the region considered, particularly near $\beta = \pi$, the equality $a_1 = a_2$ is necessary. This asymptotic expansion for the flow potential in cases $|t| \to \infty$ and $\beta \to 0$ can be written

$$\varphi \sim x + x^n g(t) C\left[B(n) + \delta \frac{g}{2g'} + \ln \frac{r}{x} + \ldots\right], \quad C = -\frac{a_1}{4}, \quad r = \sqrt{y^2 + z^2}\,, \tag{4}$$

$$B(n) = \psi(n+1) + (1-\delta)(\pi/2)\,\mathrm{ctg}\,\pi n - \ln 2 - \psi(1), \quad n \geq 0,$$

$$B(n) = \psi(-n) - (1-\delta)(\pi/2)\,\mathrm{ctg}\,\pi n - \ln 2 - \psi(1), \quad n < 0.$$

The function δ is defined as follows: $\delta = 1$ if n is an integer and $\delta = 0$ if n is not an integer. From the condition (3) and the representation (4) for $\beta \to 0$, it is necessary that $C = C_0 g'/g$ if n is an integer and $C = C_0$ if n is not an integer, where C_0 is a free constant.

For integer n, the velocity components in a cylindrical system of coordinates u_x, u_r have the following asymptotic expansion as $r/x \to 0$,

$$u_x \sim 1 + C_0 x^{n-1} g\left\{\frac{|n|}{2} + \frac{g'}{g}\left[\frac{1}{2}\,\mathrm{sign}\, n - 1 + \left(n + \frac{g''}{g'}\right)\ln \frac{r}{x}\right] + \ldots\right\}, \tag{5}$$

$$u_r \sim \frac{C_0 x^n g'}{r} + \ldots .$$

Using the formula (5) and supposing the velocity disturbance to be small in the cavity apex region and at infinity, we can obtain an asymptotic representation of the cavity shape equation as $|t| \to \infty$, according to,

$$r = \sqrt{\frac{2C_0 g'}{n+1}\, x^{(n+1)/2}}, \quad n \geq 0, \quad r = \sqrt{r_0^2 + 2C_0 g}, \quad n = -1,$$

$$r = \sqrt{r_0^2 + \frac{2C_0 g'}{n+1}\, x^{n+1}}, \quad n \leq -2,$$

where r_0 is an arbitrary constant ($r_0 \geq 0$). From the condition that the absolute value of the velocity is constant on the cavity shape, the equation for the function g(t) as $|t| \to \infty$ can be expressed as follows:

$$|n| + \frac{g'}{g}\left[\mathrm{sign}\, n\ - 2 + (n-1)\left(n + \frac{g''}{g'}\right)t + n \ln|g'| + \frac{n+1}{2}\right] = 0, \quad r_0 = 0, \tag{7}$$

$$\frac{g'}{g} = -\frac{1}{2t}, \quad r_0 \neq 0.$$

From the solution of (7), with (6), a countable set of eigensolutions can be obtained which define an asymptotic representation for the shape cavity formula as $|t| \to \infty$,

$$r=\left(\frac{2C_0x}{\sqrt{\ln x}}\right)^{1/2} \quad n=0,\ x\to\infty,\ r=\sqrt{C_0}\, xe^{-1/2\sqrt{-2\ln x}},\ n=1,\ x\to 0,$$

$$r=\sqrt{\frac{2C_0}{n^2-1}}\, x^{(n+1)/2}\,(-\ln x)^{-n/[2(n-1)]},\ n\geq 2,\ x\to 0,$$

$$r=r_0\left(1+\frac{C_0}{r_0^2\sqrt{\ln x}}\right),\ n=-1,\ x\to\infty,\ r_0\neq 0, \tag{8}$$

$$r=r_0\left[1+\frac{C_0}{(n^2-1)r_0^2}x^{n+1}(\ln x)^{(2-n)/(n-1)}\right],\ n\leq -2,\ x\to\infty,\ r_0\neq 0,$$

$$r=\left(\frac{2C_0}{\sqrt{\ln x}}\right)^{1/2},\ n=-1,\ x\to\infty,\ r_0=0,$$

$$r=\sqrt{\frac{2C_0}{n^2-1}}\, x^{(n+1)/2}\,(\ln x)^{(2-n)/(n-1)},\ n\leq -2,\ x\to 0,\ r_0=0.$$

The eigensolutions of the problem considered for $n\leq 0$ define the asymptotic representation of the cavity shape as $x\to\infty$. The case $n=0$ corresponds to the well-known solution of axisymmetric wake expansion (Levinson, 1946). The eigensolutions for $n\geq 1$ correspond to the case of cavity origination or cavity closing. The solution for the case $n=1$ has been obtained recently by Sychev (1989). It is easy to show that the solution of problem (1) satisfying (2), (3) does not exist when n is not an integer.

The solution set obtained in (8) (see Zoubtsov, 1990) increases the possibility of an asymptotic description of three-dimensional separated flow near a body with constant pressure zones, at large Reynolds numbers.

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SEPARATION AND HEAT TRANSFER UPSTREAM OF OBSTACLES

by

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Three-dimensional juncture flows occur in a variety of circumstances, including wing/body interactions and where turbine blades are joined to the hub. Recent experimental studies have clearly demonstrated that the end-wall boundary layer is inherently unsteady in the laminar regime for sufficiently high Reynolds number. A periodic process develops in the end-wall boundary layer that leads to the creation of necklace vortices which engirdle the obstacle and appear to dominate the dynamics of the flow in the juncture region. The vortices are created periodically through what seems to be an unsteady separation process in the end-wall boundary layer on the symmetry plane upstream of the obstacle. It is important to understand how these vortices develop and evolve, since their motion and influence are the dominant features of the juncture region. In applications where heat transfer is of interest, and especially the gas turbine, the necklace vortices are the major influence on surface heat transfer, giving rise to alternate moving regions of high and low heat transfer. Experiments indicate that the juncture region undergoes transition to turbulence well in advance of the rest of the end-wall boundary layer and that the process is due to instabilities that develop on the necklace vortices. In the fully turbulent regime, a large unsteady vortex is found wrapped around the obstacle which produces violent eruptions of the surface layer below.

In the present study, the development of the boundary-layer flow and heat transfer (for a heated end wall) is considered on the symmetry plane upstream of a circular cylinder mounted on a flat plate. As a first step in understanding the periodic motion observed in the experiments, the flow is impulsively started from rest, and the central interest is in understanding the eventual behavior of the boundary layer along the symmetry plane. This is a zone of persistent adverse pressure gradient and very quickly a spiral focus develops in what appears to be the beginning of the evolution of a three-dimensional vortex. Numerical solutions are obtained in both the Eulerian and Lagrangian formulations, and it is demonstrated that a separation singularity eventually occurs; in this process the boundary-layer fluid in the vicinity of the spiral focus is sharply concentrated in a band which progressively narrows in the streamwise direction, suggesting the development of a sharply focused eruption. Such events have been clearly observed in the experimental studies. Numerical solutions for the temperature field also show that sharp variations evolve in the surface heat transfer.

To compare and contrast the three-dimensional results with a similar two-dimensional phenomenon, the problem of boundary-layer development upstream of a two-dimensional obstacle was considered, namely a half circular cylinder mounted on a plane wall perpendicular to the flow direction. Again separation and sharp variations in surface heat transfer are found to develop upstream of the obstacle soon after the impulsive start. The separation time for the upstream boundary layer is approximately double that on the obstacle itself. The two-dimensional separation response is much stronger than that observed for the three-dimensional flow on the plane of symmetry.

ASYMPTOTIC THEORY OF VORTEX BREAKDOWN

by

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Vortex breakdown is one of the remarkable features of high Reynolds number rotating flow. It has been observed in flow over delta wings and in rotating axisymmetric pipe-flows (Van Dyke, 1982). This phenomenon is connected with the appearance (without any visible reason) of a stagnation point at the axis of symmetry in a thin viscous vortex core and the formation beyond this point of a closed bubble having a very slow reverse flow. This so-called "bubble" form of vortex breakdown (Leibovich, 1978) is to be considered here.

The investigation of a vortex breakdown by means of asymptotic analysis of the Navier-Stokes equations at high Reynolds number is based on an analogy with boundary layer separation phenomenon. Upstream of a stagnation-breakdown point, the quasi-cylindrical form of the equations of motion is valid for a flow in a thin viscous core (Hall, 1972). The radius of the core is $O(R^{-1/2})$, where $R = U_o L/\nu$ is the Reynolds number, U_o is the freestream velocity, L is some typical longitudinal length scale of the core, and ν is the kinematic viscosity.

Numerical solutions of the quasi-cylindrical equations (as for the boundary layer equations) shows that at some point a singularity occurs; the radial velocity component becomes infinite, but the longitudinal velocity at the axis of symmetry is not zero and remains positive. In general, this singularity is not removable (Trigub, 1985), so that it is impossible to continue the solution through this point, and this is physically unrealistic. This leads to the conclusion that vanishing of a velocity at the axis of symmetry is not a criterion of breakdown in the solution of the quasi-cylindrical equations.

The analysis of this paper shows that, in reality, vortex breakdown takes place in two stages. First at some point on the axis of symmetry, the total pressure approaches zero in the solution of quasi-cylindrical equations; however, the velocity on the axis remains finite and positive here. This represents a real criterion of the vortex breakdown. Then in a small region near the point of zero total pressure, a stagnation point appears. The longitudinal size of this region is of the order of the radius of viscous core, and here the full Euler equations for axisymmetric rotational vortex motion are appropriate.

It is shown also that the bubble as a whole to leading order is of spheroidal form. Its longitudinal and radial sizes are of the order of magnitudes $R^{-1/4}$ and $R^{-3/8}$ correspondingly. The form of the bubble is determined by the requirement that the variation of the pressure on the bubble surface is small and of the same order as the pressure in the reverse flow region.

The motion becomes possible due to the action of the radial pressure gradient associated with rotational motion. The slow reverse motion is determined by suction into thin viscous shear-layer along the surface of the bubble.

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ON THE NONLINEAR VORTEX-RAYLEIGH WAVE INTERACTION IN A BOUNDARY-LAYER FLOW

by

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Nonlinear interaction between instability waves and vortical mean flow has been experimentally identified as one of the major routes for laminar-turbulent transition in large Reynolds number flows. In the present theoretical work, the vortex-wave interaction (VWI) stemming from the development of 3-D Rayleigh instability modes in an otherwise 2-D boundary-layer flow is considered. The dominant mechanisms of the vortex-Rayleigh wave coupling has been revealed recently in the studies of flows with a strongly 3-D spectrum in the wave perturbations (Hall and Smith, 1991; Brown et al., 1993; Smith et al., 1993). Briefly, the 3-D forcing of the mean flow arises from the action of the Reynolds stresses in the vicinity of the critical layer. Alterations in the mean flow in turn bring about modulation in the wave amplitude through the coefficients in the Rayleigh equation. The VWI starts as a sufficiently high (but generally small in real terms) value of perturbations, when the two processes become mutually related.

Our prime interest here is in the VWI with initially weak three-dimensionality of the wave motion, which is often observed in various applications. The analysis is performed for large Reynolds number R. The large values of R imply, as usual, a multi-zoned splitting of the flow field in the normal direction and also a multi-scaled dependence on the streamwise coordinate. In particular, the wavelength of the nearly-neutral Rayleigh modes is estimated as $R^{-1/2}$ in appropriate normalized variables. The characteristic streamwise and crosswise length scales of the VWI region, $R^{-1/4}$ and $R^{-3/8}$ respectively, follow from the linear stability theory. The first of them is defined by the nonparallelism of the basic flow, while the second initiates effects of the spanwise variation in the wave amplitude. The level of the wave-pressure perturbations sufficient to provoke the nonlinear VWI is obtained as $R^{-7/16}$. The vertical structure of the solution comprises the main part of the boundary layer, the linear viscous critical layer in the middle of the flow, the narrow buffer zones surrounding the critical layer, and the near-wall Stokes layer. The final relation for the wave amplitude follows, from the match of asymptotic solutions in the various regions, in the form of a nonlinear integral/partial-differential equation with complex coefficients. The basic properties of the solutions for the amplitude equation (including secondary instability, finite-distance breakdown, receptivity phenomena, etc.) are investigated numerically and analytically.

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LAGRANGIAN COMPUTATION OF 3D UNSTEADY SEPARATION

by

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The predicted structure of three-dimensional unsteady separation has not yet been verified numerically. Based on the predicted structure, this paper explains why Lagrangian computations are needed for the verification. It also discusses various difficulties encountered in such a computation that are associated with the presence of large Courant numbers and sizable mixed derivatives. The performance of a number of ADI timestepping procedures, as well as of iterative procedures is considered analytically. It is shown that two ADI procedures, the *one* due to D'Yakonov and a 'one-factored' scheme can handle the constraints of a Lagrangian computation. Both are only first order accurate in time, but can be extended to higher order using passive extrapolation. It is shown numerically that the factorization errors in the D' Yakonov scheme can be kept within limits using an extrapolation based time step adaptation. It is further shown that two iterative procedures are suitable for a 3D Lagrangian computation; a three-dimensional extension of the scheme used by Van Dommelen and Shen, another an ADI procedure. Both allow an efficient multigrip based enhancement. The choice of the unsteady boundary layer configuration to be verified is discussed. The issues of coordinate singularities and transition to the two-dimensional case are addressed. Numerical results will be presented at the conference.

ANALYTIC METHODS FOR THE STUDY OF ADIABATIC COMPRESSION OF A GAS

by

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New classes of exact solutions of the complete non-linear equation of velocity potential for the unsteady spatial case are constructed. Using the results, the process of shock-free unlimited compression of polytropic gas, which at some initial instant has constant density and pressure and is at rest within either a prism or a cone-shaped body, is investigated. The laws of unsteady impermeable piston motion, which provide unlimited compression, are constructed. The degrees of accumulation of energy for such a compression are found. It is shown that the two-dimensional and three-dimensional processes of compression considered, in the case of easily-compressed cases with $1 < \gamma < 2$ (γ – adiabatic index), are more energetically advantageous than the processes of shockfree spherical compression, in creating very large gas densities.

ASPECTS OF TRANSITIONAL-TURBULENT SPOTS IN BOUNDARY LAYERS

by

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Nonlinear effects on the free evolution of three-dimensional disturbances are to be discussed theoretically, these disturbances having a spot-like character sufficiently far downstream of the initial disturbance.

For a very wide range of initial conditions, an inviscid initial-value formulation based on the three-dimensional unsteady Euler equations applies, in a first phase, allowing considerable analytical progress on the nonlinear side, as well as being suggested by much of the experimental evidence on turbulent spots and by engineering modeling and previous related theory. The time and length scales here are relatively short, of orders δ^*/U^*_∞, δ^*, respectively, for a free stream speed U^*_∞ and boundary-layer thickness δ^*. The large-time large-distance behavior then is associated with the two major length scales, proportional to $(\text{time})^{1/2}$ and to (time) in nondimensional terms, within the evolving spot. Within the former scale, the Euler flow exhibits a three-dimensional triple-deck-like structure, even though inviscid; within the O(time) length scale, in contrast, there are additional time-independent scales in operation as described later. Most effort has been made on describing the trailing edge of the spot, between the two length scales above. As the typical disturbance amplitude increases, nonlinear effects can first enter the reckoning in edge layers near the spot's wing-tips. The nonlinearity is mostly due to interplay between the fluctuations present and the three-dimensional mean-flow correction which varies relatively slowly. The resulting amplitude interaction points to a subsequent flooding of nonlinear effects into the middle of the spot. An interesting alternative means for nonlinear effects to first enter the reckoning, directly via the middle, leads to the same conclusion concerning nonlinear flooding. In either case, it is suggested that the fluctuation/mean-flow (vortex) interaction becomes strongly nonlinear, substantially altering the entire mean properties in particular. A new strong interaction between the short and long scales present, involving Reynolds stresses, is also identified. This interaction extends to the O(time) length scale covering the majority of the spot.

Later, on global time and length scales of orders ℓ^*/U^*_∞ and airfoil chord ℓ^* respectively, a second phase is encountered in which the short-scale-long-scale interaction takes on a distinct form. Here viscous-inviscid interplay matters substantially. As in the earlier stage, the three-dimensional nonlinear disturbances are traveling at speeds comparable with U^*_∞, so that conventional viscous linear instabilities are smaller and lag far behind, with their speeds $\ll U^*_\infty$. In this second phase the three-dimensional boundary-layer equations cover the spot over the airfoil scale, but the solution is fully coupled with that of the Euler equations locally, by means of Reynolds-stress forces acting in the former system and vorticity control in the latter system. The additional significance of viscous sublayer bursts is also to be noted, along with comments on links with experiments and with direct numerical simulations, and on further research.

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ON TWO-DIMENSIONAL, INCOMPRESSIBLE LAMINAR BOUNDARY LAYER SEPARATION NEAR AIRFOIL LEADING EDGES

by

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Numerical solutions are presented for high Reynolds number two-dimensional laminar viscous flow past a parabola at angle of attack. This geometry models flow past thin airfoils with parabolic leading-edges, in the limit as the airfoil thickness becomes small. Steady solutions are computed using an interactive boundary layer algorithm for flows past bluff bodies. Construction of the bluff body formulation will be presented. The interacting boundary layer algorithm employs the Veldman-Davis quasi-simultaneous algorithm, block inversions within the boundary layer, coupling of upper and lower surfaces of the parabola, eight embedded vertical grids and multiple Richardson extrapolations in all coordinate directions to achieve grid independence. Marginal separation solutions are computed over a range of angles-of-attack and Reynolds number. While the overall qualitative results of the classical marginal separation theory are confirmed, several contradictions in Reynolds number scaling suggest that slight alterations to the theory may be needed. These discrepancies may be due to finite Reynolds number effects or may require some modifications of the basic assumptions of marginal separation theory.

Unsteady classical boundary layer solutions are also computed for flow past a pitching parabola. The unsteady boundary layer equations are written using a nonlinear steady-unsteady decoupling which yields a semi-similar flow at the vertex of the parabola. The numerical algorithm is a fully implicit finite-difference method, second order accurate in all coordinate directions, and uses block tri-diagonal inversion. Numerical solutions are computed for flow past a uniformly pitching parabola. It is found that a regular unsteady flow reversal develops. However, well before the solution reaches the anticipated Van Dommelen singularity, high frequency oscillations are encountered. It is shown that these oscillatory solutions correspond to unstable boundary-layer modes predicted by Cowley, Hocking and Tutty (CHT). Qualitative comparisons are made with predicted CHT instability behavior. Quantitative validation of predicted wavenumber and mode shapes are made for both a finite wavenumber non-asymptotic analog of the Cowly, Hocking and Tutty theory as well as the full numerical boundary layer computations. The significance of the presence of CHT - modes in an unsteady boundary layer computation is that they render the problem ill-posed in time, in the sense that it should be impossible to obtain grid-independent computations of the unsteady boundary layer equations in the presence of flow reversal. This result strongly suggests that details of transition and turbulence should be addressed in unsteady flow separations.

ASYMPTOTIC MODEL OF RESONANT TRIAD EVOLUTION IN BOUNDARY LAYERS

by

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Resonant interaction of Tollmein-Schlichting waves can, under certain conditions, play an important role in boundary-layer transition to turbulence. The possibility of a strong connection within a triad of normal modes meeting a condition of nonlinear resonance was proposed by Raetz (1959) and theoretically supported by many authors (Craik, 1971; Zelman, 1974; Volodin and Zelman, 1978; to name a few). The general approach to this problem is based on the assumption that the amplitudes of interacting waves are so small that nonlinear corrections containing resonant terms can be included in a higher approximation. Then the condition of solvability of the second-order approximation gives a system of equations governing the triad evolution. The background on this problem and detailed discussion of some results obtained are presented by Craik (1985).

The present work is devoted to investigation of the resonant interaction at higher levels of amplitudes, at which previously developed techniques fail. The analysis is performed in the framework of free-interaction asymptotic theory, which was first used for this purpose by Smith and Stewart (1987). They examined, on rigorous mathematical grounds, the resonant-triad problem by the generally adopted method discussed briefly above. In the context of the governing system, we take the time-dependent three-dimensional boundary-layer equations with self-induced pressure controlling the flow in the lower layer of the triple-deck asymptotic scheme. The details of the deviation of this system can be obtained from in the monograph of Sychev et al. (1987).

If the initial amplitudes of the disturbance are small, the nonlinear regime occurs after a long stage of linear amplification, i.e. far downstream from the position of the lower branch of linear stability neutral curve. The movement downstream results in an increase of frequency and phase speed in the free-interaction-theory units. Therefore, in order to alleviate the problem, we apply the high-frequency limit of the theory as was proposed by Smith and Stewart (1987).

Order-of-magnitude estimates show that resonant interaction occurs between normal modes with the same phase speed. For this reason, it is suitable to represent the asymptotic model of the problem in terms of a small parameter ϵ equal to the inverse phase speed. In the frame of reference moving with the disturbances, the time t and cartesian coordinates (x, y, z) have the following scales:

$$t = (\epsilon^2 t_1, \epsilon^{2/3} t_2), \quad x = (\epsilon x_1, \epsilon^{-1/3} x_2), \quad z = (\epsilon z_2, \epsilon^{-1/2} z_2)\,,$$
$$y = (\epsilon y_0, \epsilon^{-1} y_1, \epsilon^{-1} + \epsilon^{1/3} y_2). \tag{1}$$

The asymptotic structure of the solution is shown schematically in Figure 1. There are three characteristic regions in the y-direction: (i) the main inviscid region where $y_1 = O(1)$, (ii) the Stokes layer where $y_0 = O(1)$, and (iii) the critical layer where $y_2 = O(1)$ and where inertial and viscous forces are comparable.

In agreement with the above consideration, the phase speed is constant at leading order and hence the solution does not depend on the "fast" time t_1. The nonlinear interaction leads to the growth of disturbances on "slow" scales, the order of which is determined from the condition that the motion within the critical layer is unsteady with respect to the "slow" time t_2, i.e. that unsteady effects caused by the nonlinearity are essential to leading order. This condition also predicts the order of the typical pressure amplitude as $O(\epsilon^{10/3})$. The solution for the pressure is expanded in an asymptotic power series of two small parameters $\epsilon^{4/3}$ and ϵ^2 produced by the critical- and wall-layer contributions, respectively,

$$P(x_1, z_1, x_2, z_2, t_2) = \epsilon^{10/3}\,[P_1 + \epsilon^{4/3}P_2 + \epsilon^2 P_3 + \epsilon^{8/3}P_4 + \ldots].$$

This representation also uniquely determines an asymptotic expansion for the velocity components. Substituting it into the main system, we obtain a number of problems for the terms of the expansion which are reduced to a successive set of integro-differential equations for the pressure functions $P_j (j = 1, 2, \ldots)$. In the main approximation we have

$$P_1 = -\frac{1}{2\pi} \iint_{-\infty}^{+\infty} \Delta P_1 \left(\xi, \zeta, x_2, z_2, t_2\right) \cdot \frac{d\xi\, d\zeta}{\left[(x_1 - \xi)^2 + (z_1 - \zeta)^2\right]^{1/2}}. \tag{2}$$

The solution of (2) can be written as a superposition of eigenfunctions corresponding to T-S waves with various orientations,

$$P_1 = \sum_{\phi \in \{\phi_j\}} A\left(x_2, z_2, t_2, \phi\right) \cdot e^{iS(\phi)}, \qquad S = x_1 \cos\phi + z_1 \sin\phi. \tag{3}$$

The amplitude function A is determined after analysis of higher approximations. The problem for P_2 is similar to (2), but contains a forced term incorporating linear and quadratic products of the first-order function P_1. The condition of solvability of the second-order problem yields the form of equations controlling the behavior of the amplitude function A. The analysis of these equations shows that the general solution disintegrates, with separate triads $\{A(\phi_0), A(\phi_0 \pm \pi/3\}$, $(|\phi_0| < \pi/6)$ developing independently of each other. Moreover, the ϕ_0-component of a triad does not undergo the influence of the other two components, so that the triad evolution is essentially linear. Hence the resonant-triad problem in the amplitude range of interest here is equivalent to an investigation of the secondary instability of the boundary layer with a superimposed T-S wave, as proposed by Herbert (1984). For example, consider the triad with $\phi_0 = 0$. Denote $A(\phi_0) = A_0$, $A(\pm\pi/3) = A_\pm$. The evolution of such a triad is governed by the following system of equations,

$$\left[\frac{\partial}{\partial t_2}+\frac{\partial}{\partial x_2}\right]A_0=0\ ,$$

$$\left[\frac{\partial}{\partial t_2}+\frac{1}{4}\frac{\partial}{\partial x_2}\pm\frac{\sqrt{3}}{4}\frac{\partial}{\partial z_2}\right]A_{+,-}= \tag{4}$$

$$\pm\frac{3\pi i}{8}\int_{-\infty}^{t_2}\left(t_2-t_2'\right)^2\exp\left(-\frac{1}{6}\left(t_2-t_2'\right)^3,\ A_0\left(t_2'\right)A^*_{-,+}\left(2t_2'-t_2\right)dt_2'\ .$$

The left-hand side operator describes the downstream propagation, while the right-hand side is responsible for interactions between the waves. If the amplitude of the two-dimensional wave is constant, $A_0=$ constant and the solution for the subharmonics $A_{+,-}$ can be sought from

$$A_+=a_+\exp\left(i\alpha_1x_2+i\beta_1z_2-i\omega_1t_2\right),\ A_-=a_-\exp\left(-i\alpha_1^*x_2-i\beta_1^*z_2+i\omega_1^*t_2\right).$$

Substitution into (4) allows one to obtain the eigenrelation between the parameters $\alpha_1,\beta_1,\omega_1$,

$$\left(\omega_1-\frac{1}{4}\alpha_1\right)^2=\frac{3}{16}\beta_1^2+\left(3\pi/8\right)^2\left|A_0\right|^2\left[\int_0^\infty\xi^2\exp\left(-\frac{1}{6}\xi^3+2i\omega_1\xi\right)\cdot d\xi\right]^2.$$

Numerical calculations of the relation were conducted for exactly half-frequency subharmonics, for which $\alpha_1+\omega_1=0$ (it is taken into account that the frame of reference moves downstream with fixed velocity). The maximum spatial growth rates $-\mathrm{Im}(\alpha_1)$ are plotted in Figure 2 versus the amplitude of the two-dimensional wave. The maximum of the growth is chosen among the waves with real values of β_1. For comparison, the results of experimental measurements of Kachanov and Levchenko (1984) (converted data from Figure 23 of their work) are also shown in Figure 2. Taking into account that in this case the small parameter ϵ is close to 1/3, we can see the good agreement between the experimental and the theoretical results.

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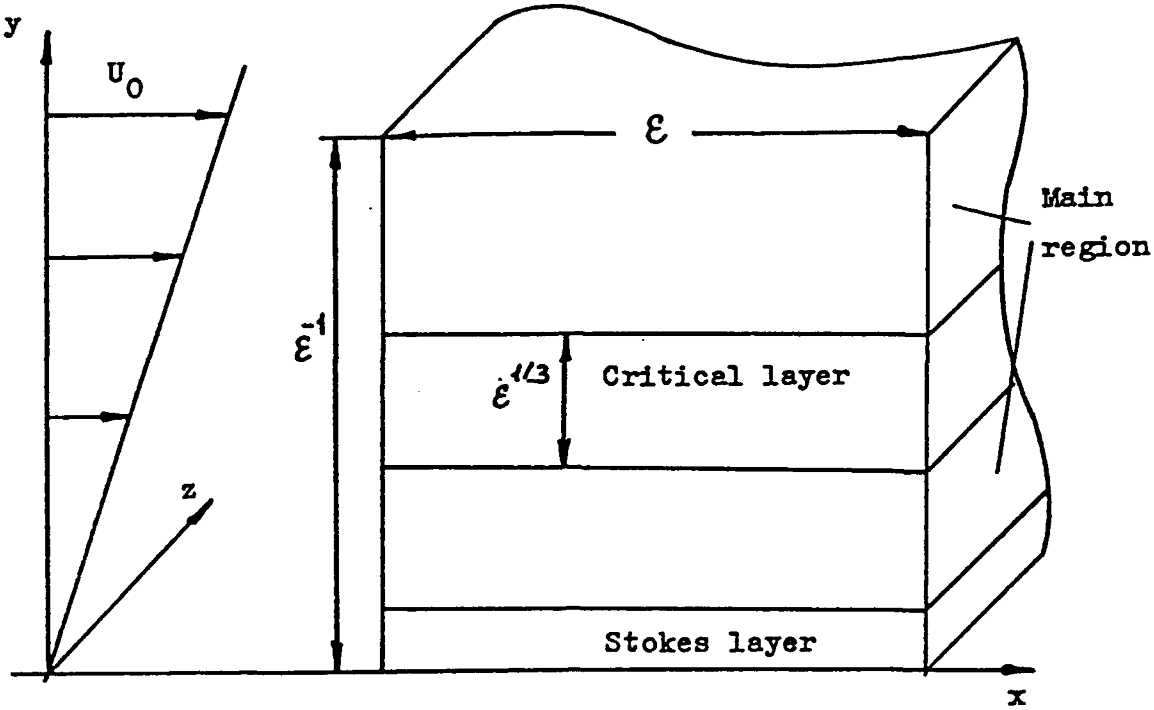

Figure 1. The structure of disturbance field within the lower layer of the triple-deck asymptotic scheme.

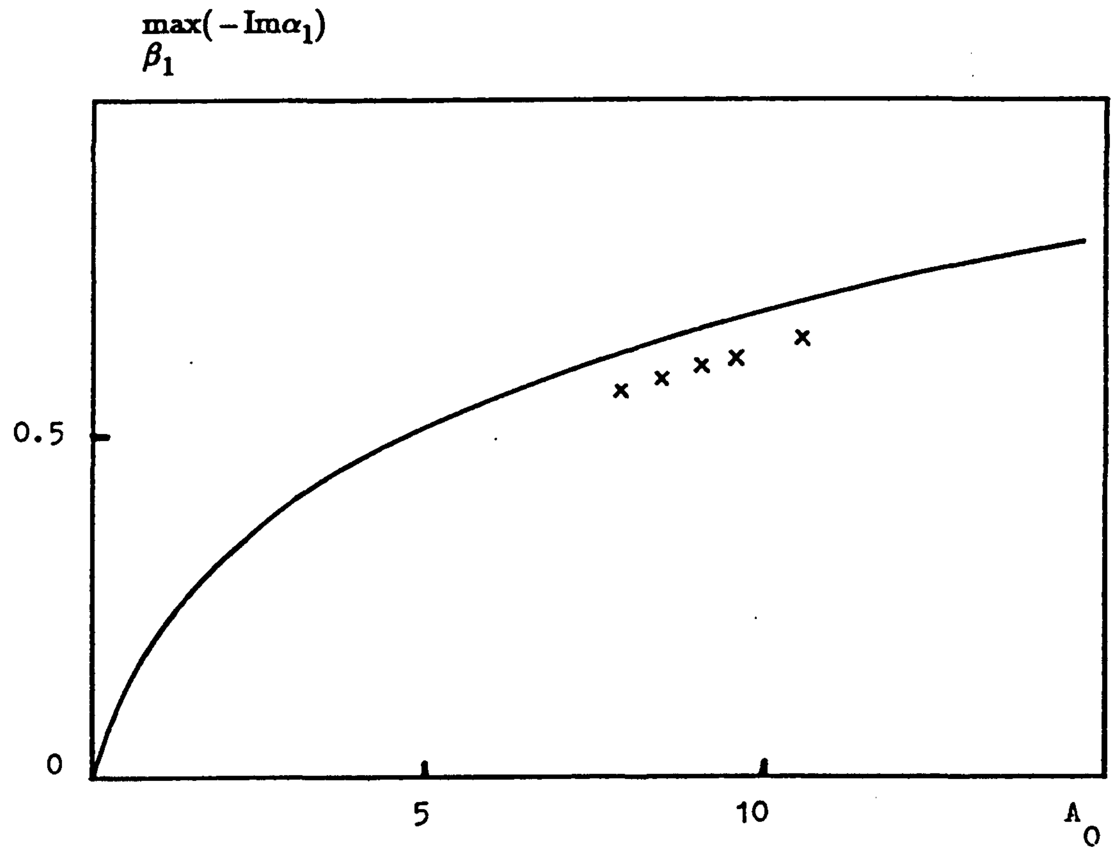

Figure 2. Maximum growth rates as a function of the amplitude of two-dimensional wave. Symbols correspond to experiments by Kachanov and Levchenko (1984).

ON DRAG AND THRUST FORCES IN AN IDEAL FLUID FLOW WITH CONSTANT VORTICITY

by

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In the class of Chaplygin flows of an ideal incompressible fluid considered here are particular viewpoints of flows past some bodies from the forces acting on them.

1. Flow with constant vorticity ω is considered in detail in an entire flow upsteam of a rectangular step. The initial velocity profile (as $x \to -\infty$, $y > 0$) is linear:

$$u = V_o + y, \quad v = 0,$$

where u,v are the velocity components along they x,y axes respectively. Here all the quantities are normalized by the step height and the vorticity value $(-\omega) > 0$. The stream function $\Psi(x,y)$ satisfies the Poisson equation with unit right-hand side and is expressed as

$$\Psi = y^2/2 + F$$

where $F(x,y)$ is a harmonic function. Using a conformal mapping of the flow region $z = x + iy$ onto the upper half-plane $t = \xi + i\eta$, and the Poisson integral, and reconstructing the harmonic function using its values on the real axis, we obtain the following exact solution for Ψ:

$$\Psi = \frac{y^2}{2} + \frac{2.65}{\pi}\eta V_o - \frac{\eta}{2\pi}\int_{-4}^{o} \frac{y^2(s)ds}{(s-\xi)^2+\eta^2}.$$

The function $y(s)$ in the integrand is known, and the mapping $z(t)$ is known as well.

Analysis of the velocity component u for $x < 0$, $y = 0$ shows that at any value of the free parameter $V_o > 0$ upstream of the step, there is a critical point $x_o < 0$ where the velocity goes to zero. It should be mentioned that on the vertical side of the step there is also a critical point $0 < y_o < 1$. At these points the fluid streamline is located which separates the unknown region of reversed flow in the vicinity of the step base from the external flow. It is natural that in the reverse flow region and in the external flow the vorticity values are equal. It is found that as $V_o \to 0$ the critical point $x_o \to -\infty$, and $y_o \to 0.639$. Without any computation, it is evident that in this case the step experiences a thrust force.

Determination of the force magnitude was performed by integrating the excess pressure (on the basis of a Bernoulli integral) along the step height. If the resulting force is normalized by the step height and the dynamic pressure of the flow at $y = 0$, $x \to -\infty$, we have the drag coefficient C_x equal to

$$C_x = \varpi - \varpi^2/3.$$

The quadratic dependence of C_x on ϖ is an exact analytical result, and the coefficients in ϖ are calculated as defined limit integrals. It is seen that at any value of the vorticity $\varpi < 0$, the step experiences a thrust force. Obviously, complete reversal of the flow results in the presence of a base drag on a backward facing step, the magnitude of which is

$$C_x = -\overline{\omega} + \overline{\omega}^2/3.$$

2. The solution for flow past a step is constructed and investigated under the assumption that in the vicinity of the base there exists a region of reversed flow with vorticity Ω, not coincident with the vorticity ω of the main flow. A stream function Ψ is represented analytically and differs from the above expression by an additional logarithmic potential of area with a given density. The streamline, separating the main stream from the reversed flow region with vorticity Ω is an integral equation solution and is determined iteratively. It is shown that at arbitrary values of ω there exist solutions, where the vorticity Ω in the recirculation zone is either greater or smaller than ω (for different values of V_o). At the same time the drag (thrust) coefficient C_x does not depend on the value of Ω and is determined by the above expression:

$$C_x = \overline{\omega} - \overline{\omega}^2/3.$$

3. The flow about a particular body is investigated, namely a finite-length wedge with an angle $\alpha = 60^\circ$, at zero angle of incidence where the freestream vorticity is

$$\omega = \omega_o \operatorname{sign}(y).$$

As in the case of the flow past a step, the stream function solution is determined explicitly using a conformal mapping z(t) of the upper half of the flow region in the z plane on the upper half-plane of the intermediate variable t. The analysis reveals that, downstream of the wedge for $\omega_o < 0$, there is always a region which is symmetric with respect to the x-axis, the spread of which along the x-axis grows as $|\omega_o|$ is increased. Upstream of the body a recirculation zone appears only if $\omega_o < \omega_{o*}$, where ω_{o*} is a certain number. As in the case of the step, the Chaplygin-Zhukovsky condition is not met at sharp edges. The force magnitude is determined by integrating the excess pressure over the entire wedge surface. The dependence of the drag coefficient on ω_o is quadratic and appears (with the base height of wedge used as a reference length) as:

$$C_x = 0.01\omega_o 0.008\omega_o^2.$$

ASYMPTOTIC ANALYSIS OF SMALL-DISTURBANCE PROPAGATION IN MIXTURES

by

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Mixtures of liquids and gasses are considered as nonhomogeneous media with periodic or stochastic structure. Let ϵ denote the ratio of the nonhomogeneity typical scale to the typical scale of the processes under consideration. If ϵ is small, the original medium equations may be approximated by averaged equations corresponding to a certain averaged homogeneous medium. The method of homogenization can be used, which is described in Bakhvalov and Panasenko (1989).

Here the averaged equations are derived and rigorously proved for mixtures characterized by certain additional small parameters besides ϵ. In particular, fluids are considered with small region γ_1 of the properly scaled viscosity and compressibility coefficients or with the small ratio λ_2 thermoconductivity and compressibility coefficients. The averaged equations for $\epsilon \rightarrow 0$, $\gamma_i \rightarrow 0$ $(i=1,2)$ are in general different depending on the relations between the orders of the small parameter values: $\epsilon \ll \gamma_i$, or $\gamma_i \ll \epsilon$, or $\epsilon \sim \gamma_i$. For $\epsilon \sim \gamma_i$, the averaged medium is a medium with memory.

For non-thermoconductive mixtures $(\gamma_2 = 0)$ with small viscosity $(\gamma_1 \rightarrow 0)$ and $\epsilon \ll \gamma_1$, the averaged homogeneous medium is isotropic independently of the original mixture structure, while for $\epsilon \ll \gamma_1$ and $\epsilon \sim \gamma_1$, the averaged medium is in general anisotropic, and the sound propagation velocity depends on its direction. This result was obtained in Sandrakov (1987) for periodic media. In Bakhvalov and Eglit (1992), it was proved that for mixtures with stochastic structure, the averaged equations for $\epsilon \ll \gamma_1$ are isotropic, and the sound velocity is equal to $(\bar{\lambda}/\bar{\rho})^{0.5}$, where $\overline{\lambda^{-1}}$, $\bar{\rho}$, are the mathematical expectations of λ^{-1}, ρ, while λ is the compressibility coefficient and ρ is the fluid density.

For inviscid fluids $(\gamma_1 = 0)$ with small thermoconductivity $(\gamma_2 \rightarrow 0)$ and periodic structure, the averaged equations when $\epsilon \rightarrow 0$, $\gamma_2 \rightarrow 0$ are, in general, anisotropic for all relations between ϵ and γ_2.

For $\gamma_2 \ll \epsilon$, heat conduction does not influence the form of the averaged equations. So as $\epsilon \rightarrow 0$, $\gamma_2 \rightarrow 0$, $\gamma_2 \ll \epsilon$, the processes may be regarded as adiabatic. But for $\gamma_2 \gg \epsilon$ the characteristic time of the processes under consideration is large enough for temperature to become uniform throughout the cell of periodicity. The sound velocities in this case are smaller than those for $\gamma_2 \ll \epsilon$ by a constant factor for all directions.

The question is also studies as to whether or not the averaged equations are still valid when properties of the mixture components (e.g. densities or viscosity coefficients) differ strongly or when the concentration of a certain component is small. Certain conditions necessary on the concentrations compressibility coefficients, density ratios are formulated for the averaged equations to be of the same form.

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THE ASYMPTOTIC THEORY OF HYPERSONIC BOUNDARY-LAYER STABILITY

by

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The linear stability of the hypersonic boundary layer is considered in the context of the local-parallel-flow approximation. It is assumed that the Prandtl number lies in the range $1/2 < \sigma < 1$ and the viscosity-temperature relation is a power law function according to $\mu/\mu_\infty = (T/T_\infty)^\omega$. An asymptotic theory in the limit $M_\infty \to \infty$ is developed.

Smith and Brown (1990) considered the case of Blasius flow and Balsa and Goldstein (1990) investigated the mixing layer; they determined that for M_∞ large, disturbances of the vorticity mode are located in the thin region between the boundary layer and the external flow referred to as the transition layer. A model gas having $\sigma = 1$, $\omega = 1$ was used in both studies. Here it is demonstrated that the vorticity mode also exists for a gas with $1/2 < \sigma <$, $\omega < 1$, but the structure and characteristics are considerably different. Nomenclature is discussed, i.e. what an acoustic mode and a vorticity mode are. Numerical solution of the inviscid instability problem for the vorticity mode are obtained for helium and compared with the solution of the complete Rayleigh equation at finite Mach numbers (Figure 1).

The limit $M_\infty \to \infty$ in the local-parallel approximation for the Blasius base flow is considered in order to understand the viscous structure of the vorticity mode. The viscous stability problem for the vorticity mode is formulated under these assumptions. The problem contains only a single similarity parameter R^*, which is a function of the Mach and Reynolds numbers M_∞, $R = (u_\infty \rho_\infty x/\mu_\infty)^{1/2}$, the temperature factor, T_f and wave inclination angle, ψ, defined by:

$$R^* = R\cos\psi C_1(T_f)\epsilon^\nu,\ \epsilon = 2/(\gamma-1)M_\infty^2,\ \nu = (1+\omega)/2\sigma + (1-\omega)/2.$$

The problem is solved numerically for helium. The function $C_1(T_f)$ is shown in Figure 2. The universal upper branch of the neutral curve obtained as a result is represented in Figure 3. The asymptotic results are compared with the numerical solutions of the complete problem (Figure 4).

In the long-wave limit, the vorticity mode starts to interact with the acoustic disturbances in the boundary-layer region. The general solution of the linear problem in the boundary-layer inner region is analyzed numerically and analytically. This solution is matched with the long-wave vorticity-mode solution near the transition layer. As a result, the inviscid instability problem for a hypersonic boundary layer is formulated. The analytical solution of this problem is found and analyzed. Different limits of the solution are considered, and the universal forms of the dependence are obtained. A similarity parameter, $s = (2\sigma - 1)/(1 + \sigma(1-\omega)1 + \omega))$, is found which is a function of the Prandtl number and the power in the viscosity-temperature law. A significant change of the solution behavior is noticed when this parameter passes a critical value of $s = 1/2$. The asymptotic structure of the amplification rate, as a function of the wave number, is described and discussed. The results are an extension of thery of Grubin and Trigub

(1993a, 1993b).

Non-parallel flow effects are also studied. It was found that the vorticity mode is highly sensitive to non-parallel effects induced by transition layer curvature. The curvature produces a centrifugal force field which influences the density fluctuations in the transition layer. (The mechanism is similar to Rayleigh instability). The asymptotic problem statement is formulated and numerical solutions are obtained for helium.

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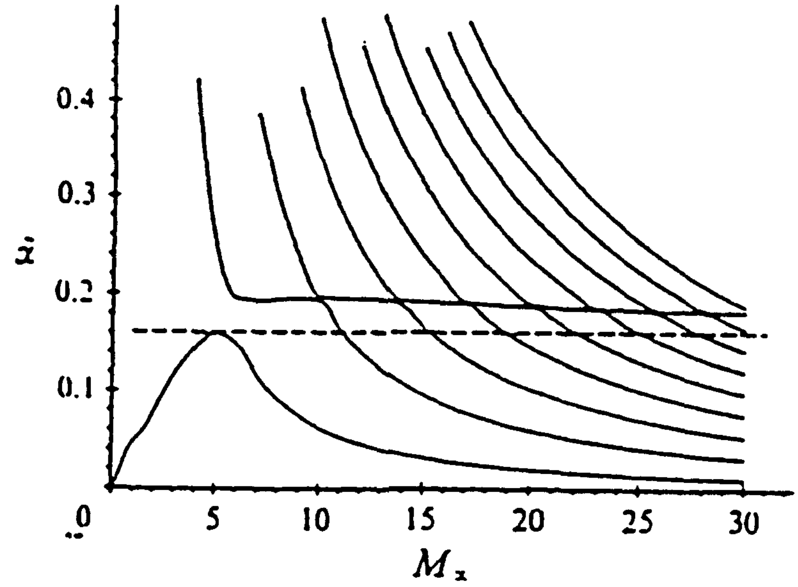

Figure 1. The neutral inflexional-mode curves at finite Mach number, $T_f = 1$. The asymptotic neutral vorticity-mode wave number $\tilde{\alpha} = 0.1603$ is shown as a dashed line.

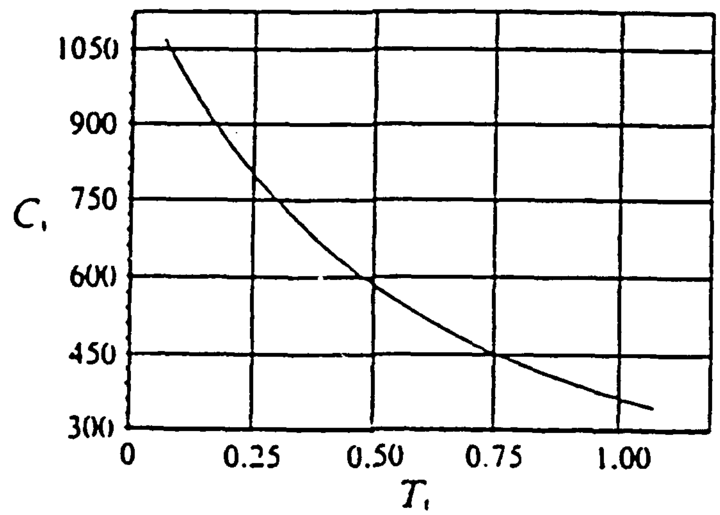

Figure 2. The dependence of the coefficient C_1 on the temperature factor $T_f = T_w/T_r C_1$ determines the intensity of the velocity defect in the transition layer: $U(y) = 1 - \epsilon^{\nu} C_f u_f$.

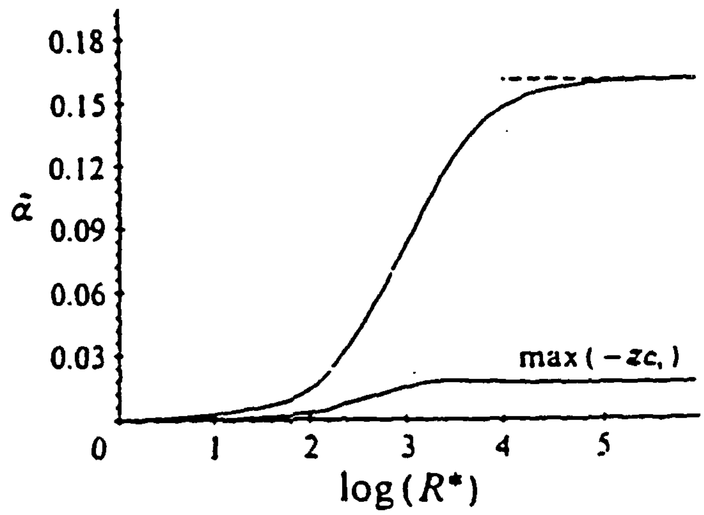

Figure 3. The universal upper branch of the neutral curves and the line of maximum amplification rate. The inviscid vorticity-mode limit is shown as a dashed line.

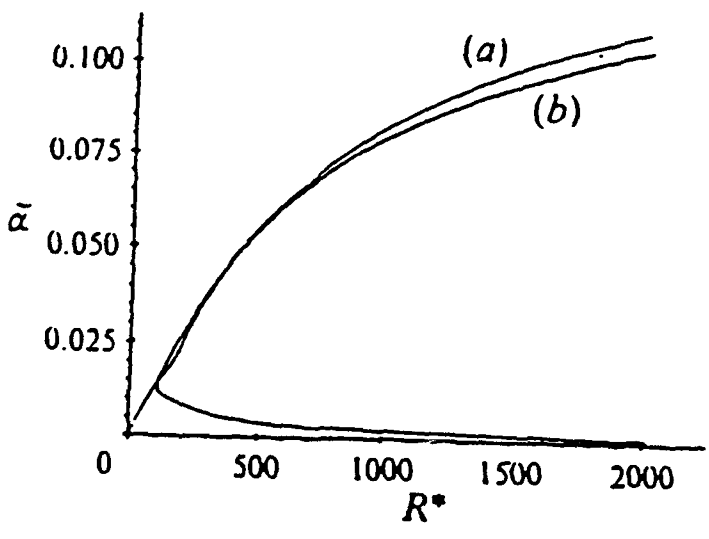

Figure 4. The universal upper branch of the neutral curves (a) and the neutral curve from the numerical solution of the complete system at $M_x = 20$, $T_f = 1$, $\psi = 0$ (b).

THIN SHOCK LAYER THEORY WITH INTERACTION: MARGINAL REGIME.

by

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This study is concerned with hypersonic flow of a perfect inviscid gas past the almost-plane butt-end of a circular cylinder, which is aligned in the direction of the stream. The analysis is carried out using asymptotic methods in the limits $M_\infty \to \infty$ and $\gamma = C_p/C_v \to 1$. It is well known (Hayes, 1959) that the shock wave in front of the cylinder is almost perpendicular (Figure 1) to the oncoming stream in these circumstances and the solution of the Euler equations is incompressible behind the shock.

Consider a small distortion of the plane butt-end surface, defined by

$$Y_w = \epsilon^{1/2} g(r, h), \qquad \epsilon = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{(\gamma+1) M_\infty^2}, \tag{1}$$

where $\epsilon \to 0$ as $M_\infty^2 \to \infty$. $\gamma \to 1$. A distortion height $O(\epsilon^{1/2})$ is sufficient to provoke a nonlinear response in the shock shape. Asymptotic expansions of solution behind the shock are expressed as follows:

$$\begin{aligned}
&v_r = \epsilon^{1/2} u_0(r, Y) + \epsilon u_1 + \ldots, \quad p = 1 + \epsilon p_0 + \epsilon^{3/2} p_1 + \ldots, \\
&v_y = \epsilon (g'(r) u_0 + v_0) + \epsilon^{3/2} (g'(r) u_1 + v_1) + \ldots, \\
&p = \epsilon^{-1} + \rho_0 + \epsilon^{1/2} p_1 \ldots, y_s = \epsilon^{1/2} f_0(r) + \epsilon f_1 + \ldots, \\
&Y = \epsilon^{-1/2} (y - \epsilon^{1/2} g(r)).
\end{aligned} \tag{2}$$

Here r is the radial variable in the plane of the butt-end and y measures distance in the normal direction. It is possible to formulate a problem for the leading order approximation $f_o(r)$ of the shock shape in the spirit of Hayes (1959), and it can be shown that f_o satisfies

$$r f_0'^2 + [f_0 - (rg)']f_0' + r = 0. \tag{3}$$

The initial condition for equation (3) does not follow from the local analysis and must be determined through external considerations. Hayes (1959) has proposed a criterion to choose a unique solution. This equation has a singular point at $f_0' = -1$ (implying the occurrence of a sonic point immediately behind the shock wave), and requiring solution smoothness at this point permits a choice of the appropriate solution. The physical sense of this additional condition is obvious; the flow near the axis is subsonic and thus the influence from the downstream flow must be considered.

The aim of this paper is to study the solution of equation (3) for different distortion shapes g(r,h), using the Hayes (1959) criterion. In particular, we consider the case where the butt-end contains a central body of variable height h. First the shape contains a central cavity defined with $h=0$; as h increases, a central body rises from the cavity. The exact studied function g(r, h) is

$$g=7r^2(1-r) + he^{-4r^2}. \tag{4}$$

It is found that at small h, the sonic point is near the cylinder corner, but as h increases, the singular point moves slightly toward the axis. It is found that the solution defined elsewhere exists within the following range of h:

$$h=[0, 2.114]. \tag{5}$$

There is a critical value of h at $h_*=2.114$. At the critical value of h, a second sonic point appears closer tothe axis of the body. From Figure 2, it may be inferred that in a small vicinity of the second singular point, the shock wave decreases linearly to $f_0' = -1$ and then rises linearly. Consequently, this is a marginal situation. Some curves for different values of h are given at the picture (Figure 2). For $h>h_*$, the solution develops a singularity which is characterized by infinite shock curvature.

It is worthwhile to study the solution of the Euler equation near the second sonic point as $h \to h_*$ to better understand a bifurcation process of solution. Consider a characteristic size δ near a sonic point with $\Delta h = h - h_*$, and consider the asymptotic structure of solution in the limits $\delta \to 0$, $\epsilon \to 0$. The pressure disturbance behind the shock is $\Delta p \sim \epsilon\delta$, it is invariant toward the surface and gives rise to $\Delta v_r \sim \epsilon\delta$ in a thin layer of potential flow. The vertical component of velocity in the main body of layer between the shock and the body has $\Delta v_y \sim \epsilon^{3/2}$, and corresponding pressure disturbance is $O(\epsilon^{3/2}/\delta)$. It is easy to see that initial pressure disturbance is comparable to an induced one when $\delta = O(\epsilon^{1/4})$, and it is necessary to take an interaction process into account. As a result we find the following interaction problem for the shock slope $\theta(R)$, and the velocity perturbation of a near wall potential jet A(R):

$$2\theta\theta'(R) - \theta - 2dR = A''(R) \tag{6}$$

$$\gamma A'' - A = 2(\theta - kR) \tag{7}$$

$$\theta = kR - \frac{\Gamma}{(-R)^{\alpha+1}} + \ldots, \qquad A \to 0, \qquad R \to -\infty \tag{8}$$

$$\theta = k_+ R + \ldots \tag{9}$$

$$A = -2(k_+ - k)R + \ldots \qquad R \to +\infty. \tag{10}$$

The parameter Γ is a scaled increment to the critical height h_*, while the constants α, d, γ, k, k_+ are known values. The problem defined permits a decreasing exponent as $R \to -\infty$; this fact means a propagation of disturbances upstream from the studied sonic point and reduces the problem to a class of interaction ones. This problem is studied numerically and a new critical value of Γ is found as a result.

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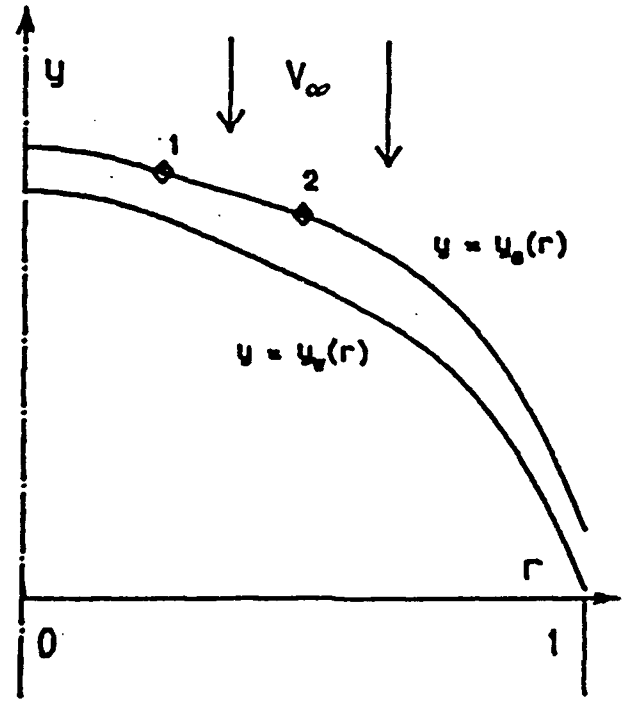

Figure 1. Scaled shape of the distortion and the shock wave; location of sonic points ($f_0' = -1$) for case 1 (h = 0) and case 2 (h = 1).

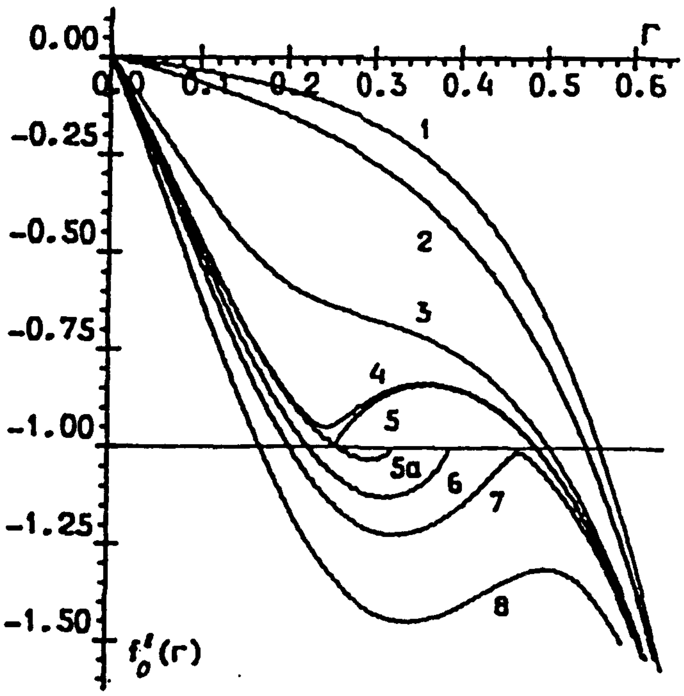

Figure 2. Profiles of $f_0'(r)$ for h = 0, 1, 2, 2.1, 2.114, 2.15, 2.192, 2.3.

UNSTEADY FLOW ON THE LEADING EDGE OF AN OSCILLATING AIRFOIL

by

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From experimental investigations in subsonic flow, it is known that in subsonic flow a small separated region (a so-called "short bubble"), forms near the leading edge of a thin airfoil at incidence. This region of reversed flow exists up to a critical value of the angle of attack α; an increase of α beyond the critical value leads to bursting of the bubble and a dramatic change in the whole flow field. Ruban (1981, 1982a) and, independently, Stewartson, Smith and Kaups (1982) have provided a theory of steady flow at the high Reynolds number which applies near the parabolic leading edge of a thin airfoil at incidence. The analysis was based on the asymptotic solution for the Navier-Stokes equations using the method of matched asymptotic expansions. It was shown that the problem of the free interaction between the boundary layer and the external inviscid flow could be reduced to the solution of the integro-differential equation

$$A^2(x) - x^2 - 2a_0 = \lambda \int_x^{\infty} \frac{A''(s)ds}{(s-x)^{1/2}}, \qquad (1)$$

for the function $A(x)$, which is proportional to the skin function; here λ is known constant. The parameter a_0 expresses a dependence of the solution on the free-stream velocity, the leading edge radius of the airfoil, and is a linear function of the angle of attack of the airfoil. Equation (1) is called the fundamental equation of the marginal separation theory.

The aim of this work is to investigate unsteady flow in the free-interaction region of the leading edge of a thin airfoil. It is known from Ruban (1982b) and Smith (1982) that such a problem can be reduced to a solution of equation which is the unsteady analog of the fundamental equation (1)

$$A^2(x,t) - x^2 - 2a(t) = \lambda \int_x^{\infty} \frac{A''(s,t)ds}{(s-x)^{1/2}} - \mu \int_{-\infty}^{x} \frac{\partial A}{\partial t} \frac{ds}{(x-s)^{1/4}}, \qquad (2)$$

where λ and μ are known constants.

A harmonic airfoil oscillation is considered with amplitude $a(t)$ given by

$$a(t) = a_0 + \sigma \cos \omega t. \qquad (3)$$

The average incidence a_0, the amplitude of oscillations σ and the frequency ω are independent parameters of the problem.

To solve the problem, two methods are considered. In the first approach, the function $A(x, t)$ is represented as a Fourier series according to

$$A(x, t) = \sum_{n=-\infty}^{\infty} A_n(x) \exp(in\omega t), \tag{4}$$

where the $A_n(x)$ are complex functions to be determined. Considering only the first m lowest harmonics ($n = 0 \ \pm 1, \ldots, \pm m$) in equation (4) and substituting into (2) together with (3), we obtain a set of nonlinear integro-differential equations for the $A_n(x)$, viz.

$$D_0 - x^2 - 2a_0 = \lambda \int_x^{\infty} \frac{A_0''(s)ds}{(s-x)^{1/2}}, \qquad n = 0$$

$$D_1 - \sigma = \lambda \int_x^{\infty} \frac{A_1''(s)ds}{(s-x)^{1/2}} - i\omega\mu \int_{-\infty}^{x} \frac{A_1(s)ds}{(x-s)^{1/4}}, \qquad n = 1$$

$$D_n = \lambda \int_x^{\infty} \frac{A_n''(s)ds}{(s-x)^{1/2}} - in\omega\mu \int_{-\infty}^{x} \frac{A_n(s)ds}{(x-s)^{1/4}}, \qquad n = 2, \ldots, m. \tag{5}$$

Here

$$D_0 = A_0^2 + 2\sum_{k=1}^{m} A_k\overline{A}_k, \qquad n = 0$$

$$D_n = 2A_0A_n \sum_{k=1}^{n-1} A_kA_{n-k} + 2\sum_{k=1}^{m-n} A_{k+n}, \qquad n = 1, \ldots, m$$

where $\overline{A}_k(x)$ is the complex conjugate function of $A_k(x)$.

The system (5) is solved numerically. The infinite limits in the integrals are by finite values (X_{-N}, X_{+N}), beyond which asymptotic approximations are used for the $A_n(x)$. The integrals are calculated by using the trapezium formula with a uniform step given by $\Delta x = (X_{+N} - X_{-N})/N$, except in the intervals $x \leq x \leq x + 2\Delta x$ and $x - 2\Delta x \leq s \leq x$ where the calculations are performed analytically.

This procedure results in a nonlinear complex algebraic equation system of order N x m. This system can be solved by Newton's method, but this requires a large amount of computing time. Therefore, another iteration method is proposed, where on each iteration A_0 is determined from the nonlinear equation system of order N using Newton's method, and $A_n (n \geq 1)$ are determined from the complex linear equations also, of order N. As a result the dependence of separation angle of attack a_0^s and that of bursting a_0^b on σ and ω are obtained.

The second approach considered here is aimed at a derivation of explicit formulae for a_0^s and a_0^b. To this end, an asymptotic solution of (2) is obtained for small amplitudes of oscillation σ. In this case leading order term in (4) may be expressed in the form

$$A_0(x) = A_{00}(x) + \sigma^2 A_{01}(x) + \ \ldots \tag{6}$$

and the leading-order approximation for (2) coincides with (1) and serves to determine $A_{00}(x)$. Next a linear equation for $A_1(x)$ is obtained where the perturbation is assumed to be of the order of σ. The solution may be obtained using a Newton technique. In order to determine the shift a_{01} in incidence

$$a_0 = a_{00} + \sigma^2 a_{01},$$

a quadratic equation must be solved as well.

A second objective of the work is the solution of the receptivity problem for the boundary layer on the leading edge of a thin airfoil which is subject to oscillations of the surface. In view of the simplicity of equation (2), the generation of Tollmien-Schlichting waves and their evolution in the boundary layer may be easily investigated for essentially nonparallel flow and even for the flow with separation.

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AN ASYMPTOTIC APPROACH TO VORTEX-BODY COLLISIONS†

by

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The collision of concentrated vortex structures with solid boundaries is common in many problems of technical interest. The focus of the present work is on the initial stages of the collision of a rotor-tip-vortex with a helicopter airframe. Based on a combination of experimental and analytical/computational work, a coherent view of the initial stages of the collision process may be identified. From the tip vortex motion, it is shown that modifications to the colliding vortex structure must occur locally near the impact point due to the development of a finite radial velocity at the assumed vortex core radius. Moreover, the impingement of the vortex leads to separation of the viscous flow underneath it; a reversed-flow eddy develops and grows in time and experiments show that boundary layer fluid will eventually be ejected into the mainstream.

To conform to the geometry of fundamental experimental measurements, the airframe is assumed to by cylindrical, and the image of the vortex in the airframe is calculated by a semi-analytical technique involving a double Fourier Transform. A typical vortex trajectory compared with experimental data is depicted in Figure 1(b). Previous results indicate that a simple Rankine core is sufficient to predict both the vortex position and the pressure on the top of the airframe in the time regime prior to impact. The impingement of the vortex induces a very strong adverse pressure gradient on the airframe under the filament which results in a very strong pressure suction peak (Figure 1(c)). Numerical results indicate that in the initial stage of the collision, the initially axisymmetric vortex core structure must be modified due to the development of a finite radial velocity at the assumed vortex core radius. This is consistent with previous experimental measurements which indicate a rapid flattening of the vortex core just prior to impact. This rapid flattening apparently begins when the vortex is still outside the boundary layer.

It is well known that computations of solutions to the Navier-Stokes equations will not normally resolve the short length and time scales which arise in the viscous flow induced near the airframe as the vortex approaches. Consequently, the boundary-layer equations are valid as the Reynolds number $Re \to \infty$ are solved. Because of the strong adverse pressure gradient induced by the vortex, the viscous flow under the vortex separates, and a complicated secondary flow develops in the form of a complicated three-dimensional eddy. The geometrical characteristics of the eddy may be elucidated through the calculation of three-dimensional streamline patterns. The viscous flow calculations completed so far are consistent with the development of a singularity in the boundary layer equations. The next step in the calculation of the viscous flow on the airframe involves the calculation of solutions to the three-dimensional interacting boundary layer equation. This work is in progress.

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STUDY OF NONSTATIONARY PROCESSES OF A STRONG VISCOUS-INVISCID INTERACTION

by

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Strong interactions between a laminar boundary layer and the external supersonic flow may arise when boundary layer separation takes place, when large disturbances influence the laminar boundary layer, and in other situations. Processes of viscous-inviscid interaction may occur in local or in global zones, with the longitudinal dimension accordingly being either much smaller or compatible with characteristic length of the body.

In this paper a review of local nonstationary viscous inviscid interaction will be given, but the main attention is devoted to studies of global nonstationary interaction processes. Processes of interaction may lead to the propagation of upstream disturbances or to the influence of downstream flow through the subsonic flow in boundary layer. Such effects may change the position of laminar-turbulent transition as well as the surface pressure and heat transfer distributions, thus altering the aerothermodynamic characteristics of an airplane.

Investigations of interaction processes conducted earlier were mainly devoted to the steady flows (Stewartson, 1955; Hayes and Probstein, 1959; Mikhailov, Neiland and Sychev, 1971), but in practice there is a need to deal with nonstationary processes of interaction. In this paper results are given for nonstationary viscous-inviscid interactions. It is supposed that the near-wall flow is described by the boundary layer equations, which include the pressure gradient induced by external or internal flow. It is important to note that this gradient is not known in advance. In this situation, the mathematical model must include an additional equation, such as that connecting the displacement thickness and induced pressure gradient. Determination of the pressure gradient for two-dimensional steady flow leads to an expression which contains the Pearson integral (Pearson, Holliday, and Smith, 1958) in the denominator. This integral diverges if the velocity on the wall equals to zero or if the wall temperature is not equal to zero. For a convergent integral, the sign determines either a subcritical or a supercritical flow regime, as defined by L. Crocco (1955).

Analysis of three-dimensional flow (Neiland, 1974) leads to the conclusion that a characteristic surface exists in the x,z plane (where x,z are curvilinear coordinates on the body surface) on which Pearson integral changes sign and where transition from subcritical to supercritical regime takes place.

In this paper the results of two-dimensional nonstationary flow studies are presented. The existence of a characteristic surface in the (x,t) plane (where x is the longitudinal coordinate and t is the time) on which an integral changes sign is demonstrated. This new integral has the form

$$\tau = \int_0^{\delta} \left[\left(\frac{a}{a_1 + u} \right)^2 \right] dy,$$

where u is dimensionless velocity in the boundary layer or in a channel, a is the dimensionless local speed of sound, y is the coordinate normal to the wall, and a_1 is the mean speed of sound. As an example, calculated results for different values of a parameter a_1 for a boundary layer, which is described by Lees-Stewartson self-similar solution (Mikhailov, Neyland and Sychev, 1971), are presented. It is found that a decrease of the temperature factor g_w leads to a decrease of parameter a_1. It is shown that properties of the boundary layer equations must be taken into consideration for developing numerical schemes.

The paper also presents numerical results of the problem pertaining to processes of interaction induced by base pressure fluctuations. It is shown that these fluctuations may influence the heat transfer distribution on surface upstream from the base edge.

A review of studies of flows in which transition from supercritical to subcritical regimes takes place is also given.

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VISCOUS EFFECTS ON CRITICAL FLOW IN THE EXIT REGION OF A THIN CHANNEL

by

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It is generally assumed that the flow velocity at the exit of a constant area tube or channel becomes sonic whenever the exit pressure is at or below its critical value. However, there have been reports that in some cases of flow through thin pipes, or capillaries, the exit flow is supersonic, so that sonic conditions must have occurred upstream of the exit plane; this would indicate the formation of a sonic throat through viscous action. Apparently, the experimental conditions were such that the flow was fully developed and laminar. The question arises as to whether this phenomenon may occur when the tube is thin, but the flow is not yet fully developed. In this paper, this possibility is investigated through the analysis of a simple channel flow.

The problem considered is that of a two-dimensional transonic channel flow with constant cross-sectional area exhausting into a plenum at subcritical pressure. The channel is thin, such that the ratio h of the half-width to the length is small compared to one. However, $h \gg \delta$, the dimensionless laminar boundary layer thickness at the channel exit; thus, an inviscid core flow with boundary layers is considered.

The flow is in the transonic regime with the Mach number being slightly subsonic at the channel entrance. The velocity component in the flow direction is written in an asymptotic expansion about sonic velocity. The Mach number increases in the channel as a result of the changes in effective wall shape represented by the displacement thickness of the boundary layer, which would nominally reach its maximum value at the exit of the channel, where the velocity perturbation would be zero. However, it is easily shown that at the sonic exit, the gradients of flow velocity and pressure for such a flow become infinite, indicating the need for a more detailed analysis in a thin (in the flow direction) region adjoining the exit plane. In this inner region, with dimensionless (with respect to channel length) thickness of order Δ, there are several layers. First is the central or core flow layer, in which the flow is inviscid. In this layer there are two possible formulations, the first of which the perturbation in flow velocity is one dimensional, while for the second it is governed by the nonlinear small disturbance equation for transonic flow and is thus two dimensional. The magnitude of Δ for the first case is larger than its value in the second and so the first case is considered; thus another, thinner inner region may be needed at the throat, or if a shock wave is formed. Next, the boundary layer divides into the familiar inviscid rotational flow layer and the viscous sublayer at the wall. Solutions for the needed orders of approximation for the velocity components and pressure in these two layers are those found for other transonic flow interaction problems. The difference here is found in the final matching with the core flow solution. First, the orders of magnitude of h and Δ and indeed all the gauge factors for the velocity components and pressure perturbations are found in terms of the Reynolds number Re. In addition one obtains a nonlinear integro-differential equation for the perturbation in velocity (and thus pressure).

It appears that this equation allows for the possibility of the velocity perturbation to change sign from negative to positive, and thus for a sonic throat to form upstream of the channel exit, with supersonic exit flow. Numerical computations under way will indicate whether such is the case, or whether only critical conditions can occur at the exit. In either event, the solution gives the proper velocity and pressure variations in the region of the exit, with no singular behavior in the functions or their derivatives.

RESONANT INTERACTIONS AND SOLITONS IN INLET PIPE FLOW

by

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The development of axisymmetric disturbances in the entrance region of a circular pipe is studied in the limit of the Reynolds number $R \to \infty$ in the framework of triple-deck theory. The following types of disturbances are considered:

(I) Lower-branch disturbances are governed by the following system of Prandtl equations (Smith, 1976; Smith and Bodonyi, 1980, Bogdanova, 1982)

$$\left\{ \begin{array}{l} \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad \frac{\partial P}{\partial Y} = 0, \\ \frac{\partial U}{\partial T} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial Y^2} \end{array} \right\} \tag{1}$$

with matching conditions

$$U - Y \to A \quad \text{as} \quad Y \to \infty, \quad U - Y \to 0 \quad \text{as} \quad X \to -\infty, \tag{2}$$

and with the pressure-displacement law

$$P = \Omega(A) \tag{3}$$

where the linear operator Ω is defined by its spectrum

$$\Omega(E) = \lambda E, \ \lambda(k) = kI_0(kr_0)/I_0'(kr_0), \quad E = \exp(ikX). \tag{4}$$

Here r_0 is the non-dimensional pipe radius, and I_0 is the modified Bessel function of the first kind and zeroth order. It is found that lower-branch disturbances can interact in a resonant manner. Numerical calculations show that a nonlinear wave packet grows much more rapidly than that in the boundary layer on a flat plate, producing a spike-like solution which seems to become singular at a finite time (Figure 1). This phenomenon is very similar to that of Peridier et al. (1991) in the structure of the forming singularity. So resonant interactions in inlet-pipe regions may provoke the bursting phenomena and lead to early laminar-turbulent transition in the whole pipe.

(II) Large-sized, short-scaled disturbances (in the lower-branch scaling) are also studied. In this case the development of disturbances is governed by the inviscid system (1) with the viscous term omitted, and the problem may be reduced to solving a single equation (Zhuk and Ryzhov, 1982; Smith and Burggraf, 1985; Rothmayer and Smith, 1987)

$$\frac{\partial A}{\partial t} + A\frac{\partial A}{\partial X} = -\frac{\partial P}{\partial X} + F(t,X), \tag{5}$$

where F is a given function representing an external source and the P-A law remains the same as given in equations (3) and (4). The homogeneous problem (3) through (5) (with $F \equiv 0$) admits *solitons*. The process of their formation under the action of external sources is studied by means of a pseudo-spectral scheme. A typical pattern is shown in Figure 2. Solitons can run both upstream and downstream depending on their amplitude. In the long- and short-wave limits, the problem in equations (3) through (5) reduces to the Korteweg-deVries and the Benjamin-Ono equations, respectively.

Such soliton behavior in the solutions is an important fact in connection with experimental observations (Borodulin and Kachanov, 1990; Kachanov, 1991), although for the case of a flat-plate boundary-layer. Detailed experiments (Borodulin and Kachanov, 1990; Kachanov, 1991) showed that soliton-like structures are formed in the K-regime of boundary-layer transition. Large-sized, short-scaled disturbances in the boundary-layer on a flat plate are governed by the Benjamin-Ono equation (Zhuk and Ryzhov, 1982; Smith and Burggraf, 1985; Rothmayer and Smith, 1987) – a classic equation which admits solitons. Detailed comparison (Ryzhov, 1990) shows that the theory (Zhuk and Ryzhov, 1982; Smith and Burggraf, 1985; Rothmayer and Smith, 1987) yields a good description of the early stage of soliton formation (when it may be considered as two-dimensional) in experiments (Borodulin and Kachanov, 1990; Kachanov, 1991), not only qualitatively but even quantitatively.

Finally, it should be noted that such solution behavior (resonant interactions leading to spike formation and solitons) is an inherent feature in inlet-*channel* flow as well (Savenkov, 1992).

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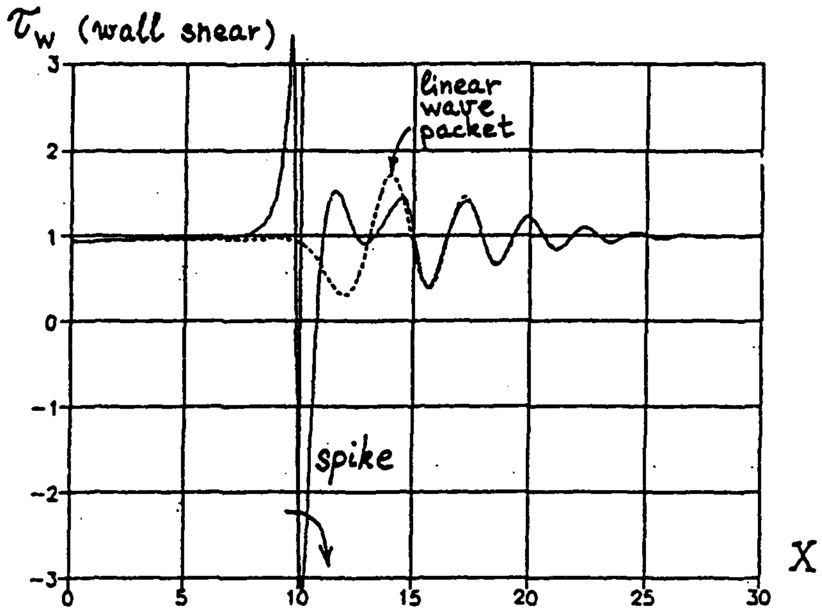

Figure 1

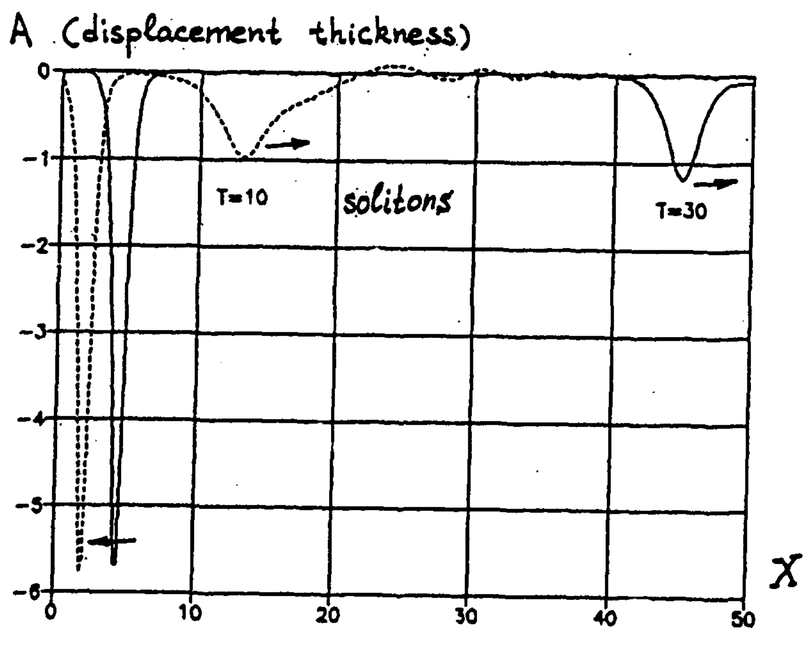

Figure 2

SOME AXISYMMETRIC TRANSONIC FLOWS

by

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Transonic flow of a gas through an axisymmetric nozzle or about an axisymmetric body is discussed. Exact potential theory and small disturbance theory are considered. Boundary value problems for the stream function and for the potential are formulated in the hodograph plane in which the equation is still nonlinear. The shape of the free streamline and sonic line are discussed for sonic and choked axisymmetric jet flow.

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ON SOLUTIONS OF BOTH THE EULER AND THE NAVIER-STOKES EQUATIONS

by

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In Cartesian coordinates (x, y), the Euler equations

$$\Delta\psi = \omega, \tag{1}$$

$$D(\omega,\psi)/D(x, y) = \delta, \qquad \delta = 0 \tag{2}$$

describe two-dimensional steady flows in terms of the stream functions ψ and the vorticity ω. When $\delta = R^{-1}\Delta\omega$, equations (1), (2) are the Navier-Stokes equations with R being the Reynolds number. In the particular case

$$\Delta\omega = 0, \tag{3}$$

the viscous flows do not depend on the Reynolds number. At the same time they satisfy the Euler equations (1) and (2). Over-definition of the systems (1)-(3) creates the hope of finding some classes of solutions and investigating all the viscous flows independent of R. Integration of the system (1) – (3) can be performed as follows. Equation (2) admits the two possibilities:

$$\psi = \psi(\omega), \tag{4}$$

or

$$\omega = c = \text{constant}. \tag{5}$$

Consider the first possibility. Substitution of (4) into (1) gives an equation of second order and equation (3) simplifies this relation to the form

$$(\omega_x^2 + \omega_y^2)\psi''(\omega) = \omega. \tag{6}$$

This equation includes two unknown functions $\omega(x, y)$ and $\psi(\omega)$. Introducing the new functions f and F,

$$f'(\omega) = [\psi''(\omega)/\omega]^{1/2}, \quad F(x, y) = f[\omega(x, y)]\,, \tag{7}$$

allow (6) to be transformed to an equation with one unknown function

$$F_x^2 + F_y^2 = 1. \tag{8}$$

The general integral of equation (8) is known while the second of equations (7) shows that

$$\omega = \Omega(F). \tag{9}$$

Substitution of equation (9) into equation (3) gives a connection between Ω and F and equation (8) simplifies it to the form

$$\Omega''(F) + \Omega'(F)\Delta F = 0. \tag{10}$$

Equation (10) has sense if ΔF depends only on F, or $H \equiv D(F, F)/D(x, y) = 0$. The general integral of equation (8) allows two possible functions F. The reverse way via equations (10), (9), and (7) gives, for one F, a general Poiseuille flow and for another F gives a new class of flows, which includes the known flow between two rotating cylinders. Thus the first possibility (4) has been exhausted.

The second possibility (5) is well-known. The functions satisfy the equations (2) and (3). The general integral of (1) when $\omega = c$ is

$$\psi = \phi(x, y) + \frac{c}{2}x^2, \qquad \Delta\phi = 0.$$

A function ψ with any harmonic function ϕ gives a solution of (1). Three boundary-value problems have been solved. Each solution represents flow with zero velocity along ellipse or a parabola or a hyperbola. On each of the curves $\psi_x = \psi_y = 0$. This leads to Cauchy problems for the Laplace equation $\Delta\phi = 0$ with the conditions $\phi_x = -cx$, $\phi_y = 0$ on the above curves. All the solutions have been found in explicit forms.

BOUNDARY-LAYER MODELS FOR FLOW PAST A CYLINDER

by

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In this presentation some models, based on boundary-layer theory, are described for determining the two-dimensional flow of a viscous, incompressible fluid past a cylinder. They are intended for application mainly to steady-state flow but can be adapted to unsteady flow in some cases; this will be discussed if time permits. A common feature of the models is that they employ the stream function and vorticity as dependent variables. Thus, whereas it is common to employ the stream function in boundary-layer analysis, it is less usual to make use of the vorticity directly as a variable.

It is well known that solutions of the boundary-layer equations for steady flow past a cylinder break down at the point of separation when the external flow is specified as potential flow, due to the appearance of the Goldstein singularity. This difficulty is discussed in detail in the present work and an interacting model is evolved in which the external flow is, in part anyway, determined from the boundary-layer solution. This is achieved by means of the use of one of Green's identities, in which the velocity distribution at the edge of the boundary layer is calculated from integrals involving the vorticity over the inner flow field. The method is an application of a method proposed by Dennis and Quartapelle (1989) for the Navier-Stokes equations, although there are difficulties in applying it to a boundary-layer model of steady-state flow. The reason is that the Green's identity requires that the vorticity shall be known throughout the whole inner field of flow, whereas the boundary-layer solution may provide it only in part of the field; nevertheless, some progress can be made. Moreover, in some cases of unsteady flow, e.g. those of a cylinder started from rest, the problem is in some senses simpler since the velocity at the edge of the boundary-layer can be calculated as part of the solution.

By means of the interacting method, it is possible to integrate the boundary-layer equations beyond the point of separation for steady-state flow. However, the integration becomes more difficult in the separated region, and this leads to the development of an improved model of the boundary-layer equations, which is achieved by making a prior transformation of the vorticity to a new dependent variable. This transformation attempts to take into account the asymptotic features of the wake at large distances from the cylinder, while at the same time conforming to the boundary-layer structure near the cylinder. This model can be integrated beyond the point of separation. Several interacting models are discussed, one of which utilizes the Green's identity.

Calculations have been performed for the case of flow past a circular cylinder and comparisons are made with the standard boundary-layer model (Schlichting, 1979, p. 171). However, the models are applicable to any cylindrical cross-section if a suitable mapping is use.

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HIGH-REYNOLDS-NUMBER STRUCTURE OF STEADY TWO-DIMENSIONAL FLOW THROUGH A ROW OF BLUFF BODIES

by

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An extension of an earlier theory (Chernyshenko, 1988) *of flow past an isolated body is described. For a crossflow cascade of bodies a distance 2H apart, the region of validity of the extended theory covers $H \gg 1$. A comparison with numerical calculations is favorable.*

Steady flows around a cascade of bluff bodies have recently attracted considerable attention. For example, theoretical investigations were carried out by Smith (1985) and Milos and Acrivos (1986), and numerical calculations have been reported by Milos, Acrivos and Kim (1987), Ingham, Tang and Morton (1990), Fornberg (1991) and Natarajan, Fornberg and Acrivos (1992). The latest review of the subject has been written by Fornberg (1992).

A high-Re asymptote of the two-dimensional steady solution to the Navier-Stokes equations for the incompressible flow through an infinite row of bluff bodies located at equal distances 2h across the flow, is found. All quantities are normalized with respect to the velocity at infinity, the fluid density, and the characteristic size of the body. All bodies are of the same shape and size and possess a symmetry axis parallel to the undisturbed flow direction. The general sketch of the flow is shown in Figure 1.

The asymptotics for $Re \to \infty$ depend on the behavior of H as a function of Re. Assume that for sufficiently large Re the ratio H/L (where L is the eddy length) is fixed. When L(Re) is found, the resulting asymptotics may be considered as asymptotics for a given H(Re).

Only the main points of the theory will be given here. Suppose that the eddy grows indefinitely with increasing Re, but with the eddy length and width being of the same order. Then on the eddy scale, the body shrinks to a point and the flow tends to the inviscid flow consisting of a cascade of touching pairs of symmetric closed-streamline regions of constant vorticity (by the Prandtl-Batchelor theorem) of equal values and opposite signs. Outside these regions, the flow is potential. It has been proved (Chernyshenko, 1988) that in the correct asymptotic limit, the jump in the Bernoulli constant across the eddy boundary is zero. Such inviscid flows were calculated by Chernyshenko (1991) and for the case of an isolated pair of vortex regions by Sadovskii (1970) and by Saffman and Tanveer (1982). For a zero jump in the Bernoulli constant and a given velocity at infinity, these Sadovskii flows are uniquely determined by L and H. Hence the vorticity in the eddy ω_∞ is related to L by the formula

$$\omega_\infty L = C_1(H/L), \tag{1}$$

where C_1 can be found from numerical calculations.

According to the Bobylev-Forsythe theorem (Serrin, 1959), the rate of energy dissipation is proportional to the vorticity square integrated over the entire flow field. In Sadovskii flow the vorticity is non-zero only inside the eddy. Hence the rate of energy dissipation is proportional to $\omega_\infty^2 L^2$ and the rate of energy dissipation equals the product of the drag and the velocity at infinity. Provided that the contribution to the integral from the smaller regions is negligible (this can be proved when the complete structure is known), the drag coefficient is

$$c_d = \text{const}\ \omega_\infty^2 L^2/\text{Re} = \text{const}\ C_1^2/\text{Re} = C(H/L)/\text{Re}, \tag{2}$$

where $C(H/L)$ can be found from the Sadovskii flow. Here $c_d = drag/(U_\infty^2 R)$, R is the length scale and $\text{Re} = U_\infty R/\nu$. Expressing the drag via far-wake characteristics, and considering the boundary layer surrounding the eddy and the wake, gives the same result.

The vorticity balance yields the third important relation. The vorticity diffuses from the eddy and is then convected downstream in the wake. The vorticity also diffuses toward the symmetry line where it is zero. The loss of vorticity is compensated by the vorticity shedding from the body. Naturally, due to symmetry the total vorticity flux from the upper and lower parts of the body is zero. Here we consider only the part of the flow above or below the symmetry line. As the Reynolds number based on the eddy length is ReL, the thickness of the wake and boundary layer surrounding the eddy is of the order $L/\sqrt{\text{ReL}} = \sqrt{L}/\sqrt{\text{Re}}$. Hence the vorticity flux is of the order $\omega_\infty\sqrt{L}/\sqrt{\text{Re}}$. The vorticity flux to the symmetry line has the same order of magnitude. Therefore the vorticity flux F from the body is

$$F = C_3\omega_\infty\sqrt{L}/\sqrt{\text{Re}}. \tag{3}$$

Here C_3 cannot be found from the Sadovskii flow calculations alone, a careful examination of the boundary layer surrounding the eddy is necessary.

The last relation can be found from the body-scale flow, which is a Kirchhoff flow with free streamlines. The vorticity is convected downstream from the body in the mixing-layer near the free streamline. Hence the vorticity flux F is $\int u\omega dn = -\int u\frac{\partial u}{\partial n}dn = (u_-^2 - u_+^2)/1$, where u_- and u_+ are the velocities at the mixing layer boundaries and the integral is taken across the mixing layer. Inside the eddy in the Kirchhoff flow the velocity is zero. Hence the velocity on the free streamline is $u_+ = \sqrt{-2F}$. Therefore the drag coefficient is

$$c_d = k_d = u_+^2 - 2k_d F, \tag{4}$$

where k_d is the Kirchhoff drag coefficient for a free-streamline velocity equal to 1.

Now the system (1–4) allows the four unknowns L, ω_∞, c_d and F to be found easily, giving all the main characteristics of the flow. The resulting formulae can be written in the following form,

$$\omega_\infty = -2CD_0^2(b)/(k_d^2\text{Re}), \tag{5}$$

$$F = -C/(2k_d\text{Re}), \tag{6}$$

$$L = k_d^2\text{Re}/(2D_0^2(b)\sqrt{\alpha C}), \tag{7}$$

$$c_d = C/Re. \tag{8}$$

The function $D_0(b)$ here is part of the expression for C_3 following from the quantitative analysis of the vorticity balance in the recirculating layer (Chernyshenko, 1982, 1988).

b	0.1	0.2	0.3	0.5	0.7	0.9
$D_0(b)$	0.0090	0.1294	0.1619	0.2217	0.2857	0.3744

To use Chernyshenko's (1991) numerical results, C_1 and, in part, C_3 are expressed via $C = C(H/L) = \omega_\infty^2 S$, where S is the total area of both halves (upper and lower) of the eddy, $b = b(H/L)$ which is the ratio of $\int u(s)ds$ (taken along the lower eddy boundary from the forward to the rearward stagnation point) to the circulation of the velocity around the lower half of the eddy and $\alpha = \alpha(H/L) = S/L^2$.

According to (7), $L = O(Re)$ for a fixed H/L. Hence this theory yields an asymptote for $Re \to \infty$, with $H = O(Re)$. Careful examination shows it to be valid in a wider range $Re \gg 1$, $H \gg 1$.

The asymptotic structure includes other characteristic regions apart from the already mentioned eddy scale, body scale, and recirculating boundary layer. These regions provide a proper matching, ensuring self-consistency of the theory.

For comparison of the asymptotic results with numerical calculations, the vital question is: how large must Re and H be to expect a reasonable agreement? According to the theory the velocity on the free streamline near the body equals $\sqrt{-2F} = \sqrt{C/(k_d Re)}$ and therefore tends to zero. Hence good agreement can be expected only for $C/(k_d Re) \ll 1$. For the flow past an isolated body $C \approx 73$, and it is even larger for cascade flows (Chernyshenko, 1991). For this reason a good agreement could hardly be expected for the Reynolds numbers achieved so far in numerical calculations. Figure 2 shows a comparison of the eddy length given by (7) with the numerical results of Fornberg (1991) for a row of circular cylinders and of Natarajan, Fornberg and Acrivos (1992) for a row of flat plates. It is worth noting that the transition in the behavior of the numerical results as a function of Re, most clearly seen in Fornberg's results for $H = 50$, occurs approximately for $C/(k_d Re) = 1$. The comparisons of other flow characteristics are similar. The numerical results approach the theoretical ones with H or Re increasing. On the whole, it may be concluded that, although the Reynolds numbers achieved in numerical calculations were not sufficiently large to exhibit clearly an asymptotic behavior, and in the case of Natarajan, Fornberg and Acrivos' (1992) calculations, the values of H also were not sufficiently large for comparison with our theory, the tendencies observed in the numerical results support the theory.

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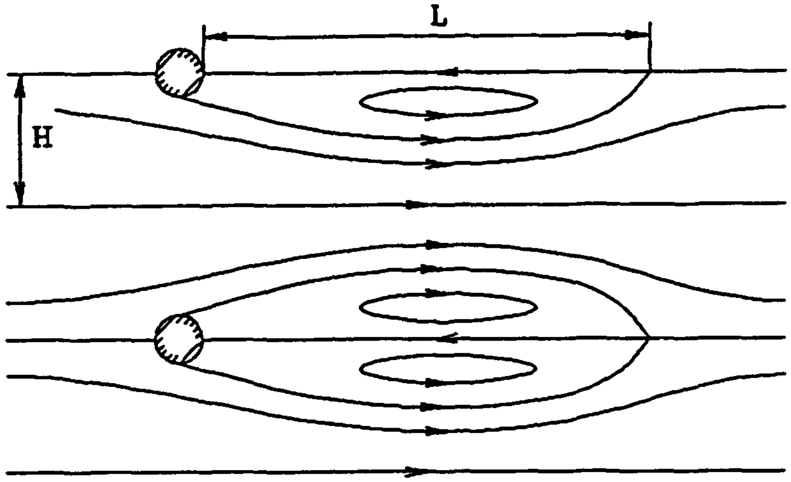

Figure 1

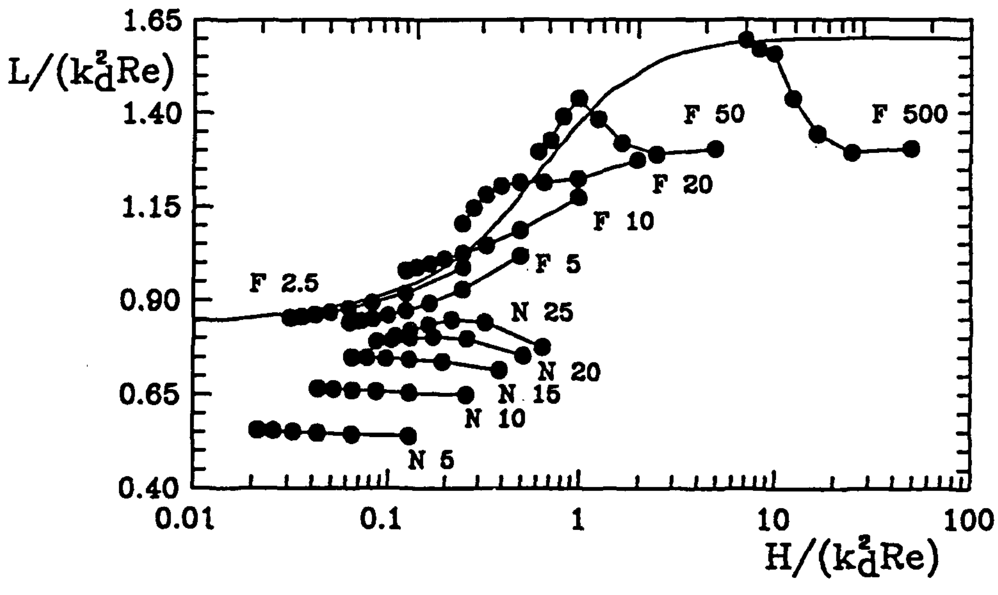

Figure 2. The eddy length. Natarajan et al. (1992) results are marked with N and those of Fornberg (1991) with F. The numbers near these letters are the values of H. The curve is the theory. For the Natarajan results, Re = 50 − 300 at intervals of 50, from right to left. For the Fornberg results, Re = 50 − 400 and for H = 500, Re = 50 − 350 at intervals of 50.

WEAKLY AND FULLY NONLINEAR EFFECTS IN CHANNEL FLOW TRANSITION: AN EXPERIMENTAL COMPARISON

by

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This work aims to identify the weakly (I) and fully nonlinear (II) processes at work in channel flow transition and to make comparisons of the predictions of the theory with the experimental results of Nishioka et al. (1979) on the transition of plane Poiseuille flow. Comparisons are also made of some aspects of the theory (II above) with experimental results on transitional separated boundary-layer flow. A "bypass" transition criteria is suggested.

The method used to approach the problem involves a scale analysis of the governing equations which leads to a nonlinear interactive boundary-layer system governing transition over relatively long scales, although (II) is also relevant to short-scale transition. Analytical progress is made possible by considering a disturbance which is of a relatively high frequency on these scalings and whose short-scales allows a weakly nonlinear analysis to be pursued (I).

This weakly nonlinear system governs the interaction between a three-dimensional wave and a small perturbation to the mean boundary layer profile which varies over relatively slow spatial and temporal scales. Similar equations have been derived by Stewart and Smith (1992) in the boundary-layer context and comparisons made with the experimental work of Klebanoff et al. (1959). These authors show that the governing equations are subject to a finite time/distance breakdown with a shortening of the spanwise scales and an increase in the amplitude of the disturbance. This is interpreted as the generation of "streets", and the predictions of the theory compare favorably with experiment. A more nonlinear stage is then reached in which the total mean flow profile is altered from a uniform shear profile and the phase speed of the wave becomes unknown. The governing equations in this stage are the full interactive inviscid boundary-layer equations, with no streamwise pressure gradient and a nonlinear forcing from the wave, allied with a viscous sublayer close at the wall which may not remain passive.

Initial indications suggest that these fully nonlinear and three-dimensional equations are subject to the finite-time breakdown described in Smith (1988) in which the streamwise pressure gradient becomes infinite, corresponding to nonlinear wave breaking and the appearance of the first "spike" in the transition process (II above). This breakdown is possible if a certain integral of the mean flow profile becomes zero as the flow develops, and this condition is proposed as a transition criteria for "bypass" transition (i.e., omitting stage I) in sufficiently strong disturbance environments. It is shown that the experimental profile measured by Nishioka et al. at the occurrence of the first "spike" satisfies this condition to within acceptable accuracy.

This test is also used in a comparison with transitional separated flow in experimentally determined velocity profiles provided by Professor M. Gaster.

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ON THE TRANSITION TO INSTABILITY IN FLOWS DEPENDING SLOWLY ON A SPATIAL VARIABLE: THE STABILITY OF PIPE FLOW

by

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Asymptotic methods are considered for constructing the linear perturbation modes which are responsible for instability of solutions that depend weakly on one spatial coordinate (i.e. depending on x/L, where L is large). Recently, in papers by Huerre and Monkewitz (1990) and Monkewitz (1990), one criterion of instability (referred to as a global stability criterion[1]) has been established and applied to some specific flows. This was found under the assumption (which seems to be rather common) that the perturbation mode yielding instability consists of two waves whose wave numbers have two very close (in the limit coinciding) turning points in the x-plane. This instability is always accompanied by fulfillment of a local criterion of absolute instability in some interval on the real x-axis. The same criterion was obtained in Iordanskii (1988) by a different method.

In earlier papers (Rukhadze and Silin, 1964; Zaslavskii, 1982; Kulikovskiy, 1985), the perturbation modes in the most common cases are shown to consist of a closed succession of waves (a chain), turning one into another. The purpose of the present article is to demonstrate a physically real example of a flow in which the appearance of instability is connected with a wave chain containing more than two waves.

The oscillation of a compliant pipe with flowing fluid may be described by the equation:

$$\rho_1\frac{\partial^2 w}{\partial t^2}+\rho_2\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)^2 w=-Fw-D\frac{\partial^4 w}{\partial x^4}.$$

Here ρ_1, ρ_2, v, F, D are slowly varying functions of x and assumed to depend on a number of parameters. It is found that a domain of parameter variations exists, in which

1. the instability manifests itself by appearance of a wave chain containing four waves;

2. everywhere on the real x-axis, the local condition of absolute instability is not valid.

[1]The term global stability was employed previously in a different sense for the class of instabilities of uniform flows with boundary conditions (Kulikovskiy, 1966; Lifshitz and Pitaevskii, 1981).

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ASYMPTOTIC EQUATIONS FOR THE BOUNDARY LAYER USING A DEFECT FORMULATION

by

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The standard laminar boundary layer theory, as first developed by Prandtl, gives accurate predictions when the boundary layer thickness is sufficiently small, so that the inviscid flow can be assumed to be constant across the boundary layer. But when the inviscid flow variations cannot be neglected, the matching of the boundary layer with the outer flow is no longer ensured, since the standard matching procedure only involves the inviscid flow values at the wall (Figure 1 – left panel). This occurs for strongly sheared inviscid flows, such as the shock layer along a hypersonic blunt body. To deal with this phenomenon, in 1962 Van Dyke built an extended theory, called higher order boundary layer theory, relying on matched asymptotic expansions, and embedding Prandtl equations. Thereby he verified several second order effects such as wall curvature, velocity gradient, or displacement, and showed their influence on the global coefficients like the skin friction or the wall heat flux. When limited to second order expansions, the matching is improved (Figure 1 – right panel) but is not yet fully satisfactory if the inviscid profiles are not linear.

To ensure a smooth matching whatever the inviscid profiles, we propose to use a defect formulation for the boundary layer, together with matched asymptotic expansions. In the wall region, the variables dealt with are no longer the physical variables but the difference between the viscous solution and the outer inviscid profile, labeled E in Figure 2. These new variables called defect variables and labeled D; they are expanded in powers of a small parameter $\epsilon = 1/\sqrt{Re}$, where Re is the Reynolds number. A stretched normal coordinate $\bar{y} = y/\epsilon$ is also introduced, as in the Van Dyke theory. The expansions for the outer region are identical to Van Dyke's. Then these expansions are substituted into Navier-Stokes equations, and the terms of like powers in ϵ are equated, giving a new system of boundary layer equations; the properties of these equations are quite similar to Van Dyke's apart from the matching, which is obtained by letting the defect variables tend to zero outside of the boundary layer. Thus the defect solution merges smoothly into the external inviscid flow even with a first order expansion. These equations are parabolic and can be solved using fast space marching methods, as long as the boundary layer remains attached.

The defect equations have been first derived for incompressible laminar flows, and particular attention has been paid to self-similar solutions for a constant shear flow past a flat plate, which emphasize the differences and analogies between the standard and defect approaches. For this particular case, the first order defect solution is seen to stand in an intermediate position between Van Dyke's first and second order solutions. Then, a two-dimensional compressible flow has been considered. The defect boundary layer equations have been written and solved along plane or axisymmetric hypersonic hyperboloids. When compared with the standard boundary layer (Figure 3), the defect solutions generally show a closer agreement with Navier-Stokes solutions, allowing a better prediction of the skin friction and the wall heat flux, for a similar cost.

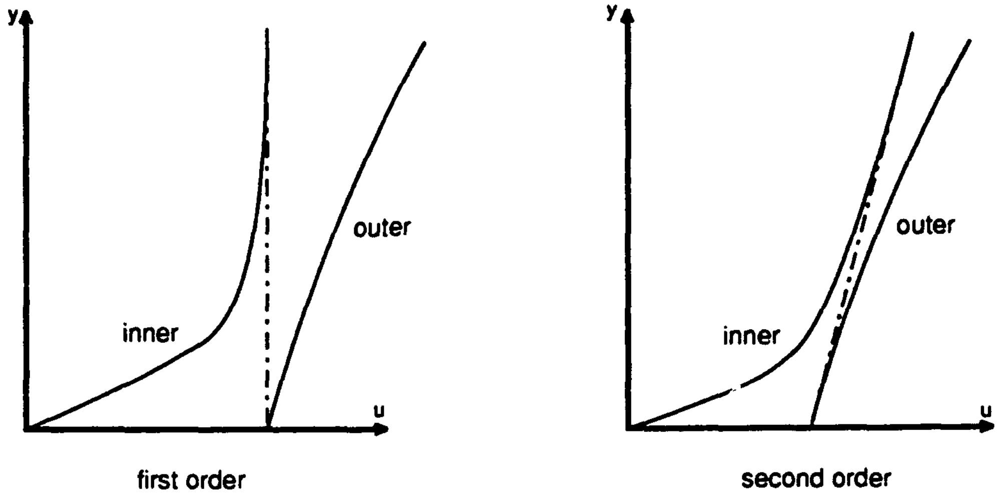

Figure 1: matching schemes

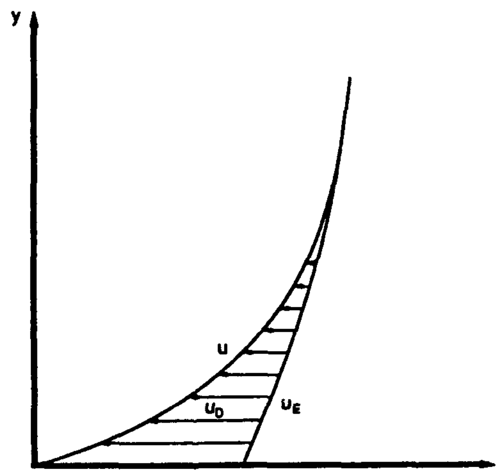

Figure 2: defect velocity

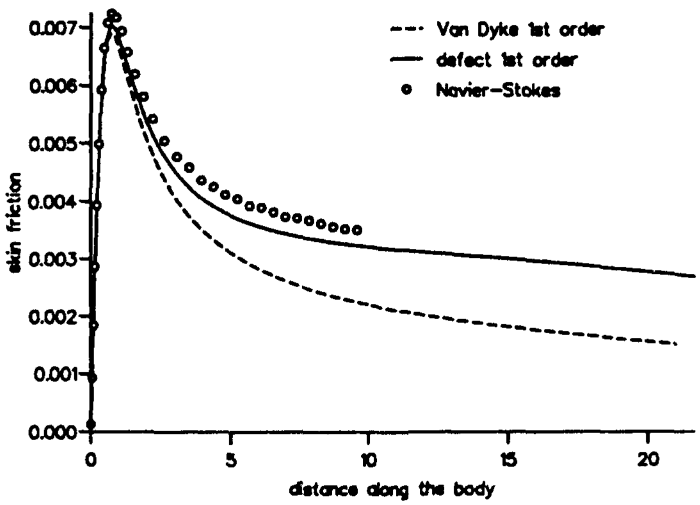

Figure 3: Skin friction on the hyperboloid - Mach 23.4, T_w = 1500 K

ASYMPTOTIC STUDY OF DISSIPATION AND BREAKDOWN OF A WING-TIP VORTEX

by

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The steady, axisymmetrical wing-tip vortex is studied in this paper by means of asymptotic methods in the limit of high Reynolds numbers. The smooth regrouping of the vortex under the action of viscous forces is described by a quasi-cylindrical approximation.

The initial conditions were chosen from the two-parametric class of velocity and circulation profiles:

$$u^o = \alpha + (1-\alpha)e^{-y}, \quad g^o = g_{\infty}(1-e^{-y}), \quad y = (r/r_o)^2/2.$$

The velocity and circulation distribution upstream from the position of a vortex breakdown (Leibovich, 1978) are approximated well by such profiles.

The solutions of the quasi-cylindrical approximation are thoroughly analyzed numerically, and it is shown that saddle point bifurcation appears at certain critical values of circulation. At these values the solution may be continued in two ways; as a supercritical branch which approaches Batchelor's limit far downstream and a subcritical one, which passes the second, nodal-point bifurcation.

The problem of continuation for the quasi-cylindrical approximation with the initial conditions stated at the nodal point does not have a unique solution but, instead, an infinite one-parameter family of the solutions controlled by an arbitrary constant c_A. The problem is not correct beyond the second bifurcation point, and an additional downstream condition must be specified to provide uniqueness of the solution. The parabolic equations of the quasi-cylindrical approximation allow the downstream disturbances to propagate upstream.

A similar situation is known in the problem describing of hypersonic boundary layer on a flat plate in the strong interaction regime. It was discovered by Neiland (1970) (and extended by Brown, Stewartson and Williams (1975)) that an eigenfunction $cx^k f(\eta)$, where x is the distance from the leading edge, η is a self-similar variable, c is an arbitrary constant and $k > 1$, appears in the asymptotic expansions of the solution near the leading edge, which is the singular point for the problems of interest. The solution of the parabolic boundary layer equations is not unique and one downstream condition for a scalar quantity must be added. Usually this condition is stated for the downstream pressure. It is possible to change the solution in the whole interval from the leading edge to the last downstream position by changing the downstream pressure. Therefore, upstream propagation of the downstream disturbance exists.

The flow past the second bifurcation point was studied numerically, and it was shown that the solution of the quasi-cylindrical approximation with large reversed flow regions exist. Results for eight solutions obtained for $x_o = 0.1$, $x_1 = 0.3$, 0.32, 0.35, 0.4, 0.45, 0.55. 0.8, and 1.0 are shown in Figure 2. An important feature is that the solutions on the front portions of intervals have a weak sensitivity to changes in the downstream conditions. The solutions 1, 2, and 3 were obtained with additional downstream conditions:

$$\frac{du}{dx}(x_1,0) = 0.$$

The asymptotic expansion of such solutions far downstream was constructed, and it emerges that the reversed flow region expands exponentially. This process is halted by elliptical effects in the external flow. An asymptotic theory for large reversed flow regions is suggested including viscosity and elliptical effects. Numerical solutions for unbounded vortex breakdown parabolically expanding far downstream are presented in Figure 3. The values $f^{1/2}$ and x correspond to the specially scaled radius of the recirculation zone and axial coordinate. Solutions 1, 2, 3 are terminated at the singular points; 5, 6 correspond to a linearly expanding zone and an intermediate solution 4 to the parabolically expanding one.

The general asymptotic problem statement which describes the flow near the bifurcation points is used to study the asymptotic solutions near the first bifurcation point. The problem is investigated numerically and two kinds of solutions, which may be treated as transcritical jumps and marginal vortex breakdown, are found. A number of solutions corresponding to marginal vortex breakdown are presented in Figure 4 where distributions of the disturbance of axial velocity are shown. The curves 1-5 are close to singular solutions of the quasi-cylindrical approximation far upstream. The curve 6, for example, is the singular solution of the quasi-cylindrical approximation which coincide far upstream with solution 5. The elliptical effects in the vicinity of the saddle bifurcation point remove the singularity and permit finding the solution which tend to Batchelor's limit far downstream.

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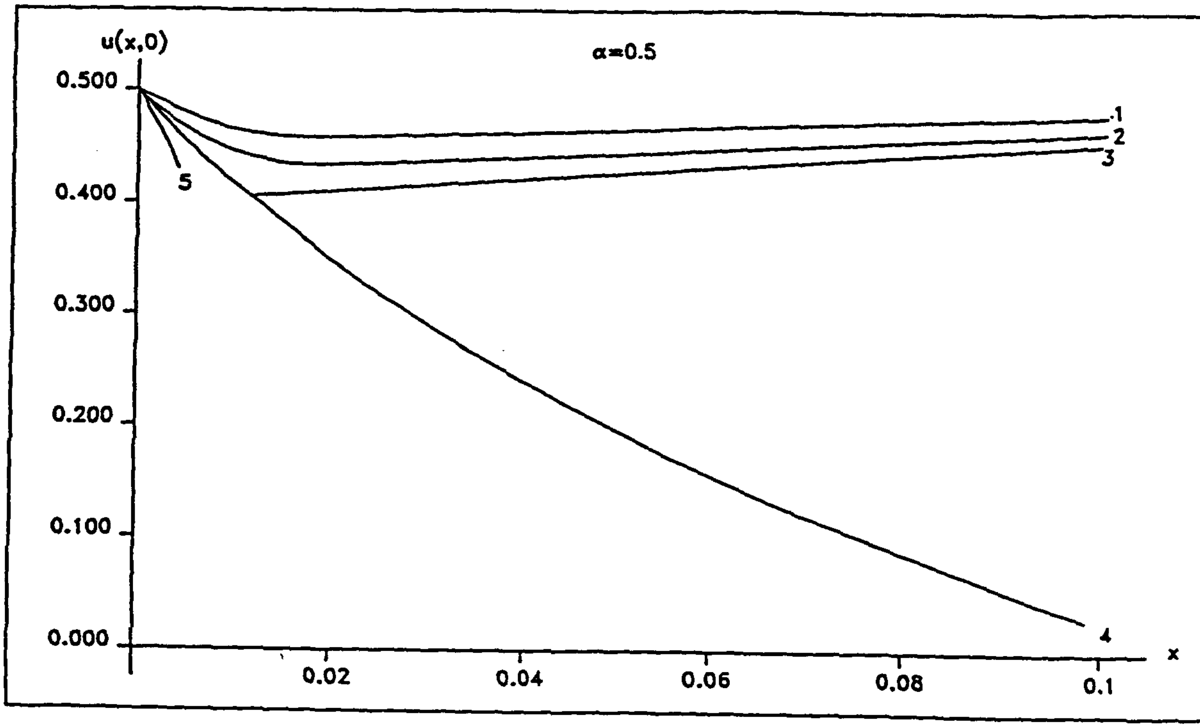

Figure 1: The axial velocity on the axis u(x,0) at $\alpha = 0.5$. The lines 1-5 correspond to $g_\infty = 1.14$, 1.17, 1.18562, 1.185624, 1.2.

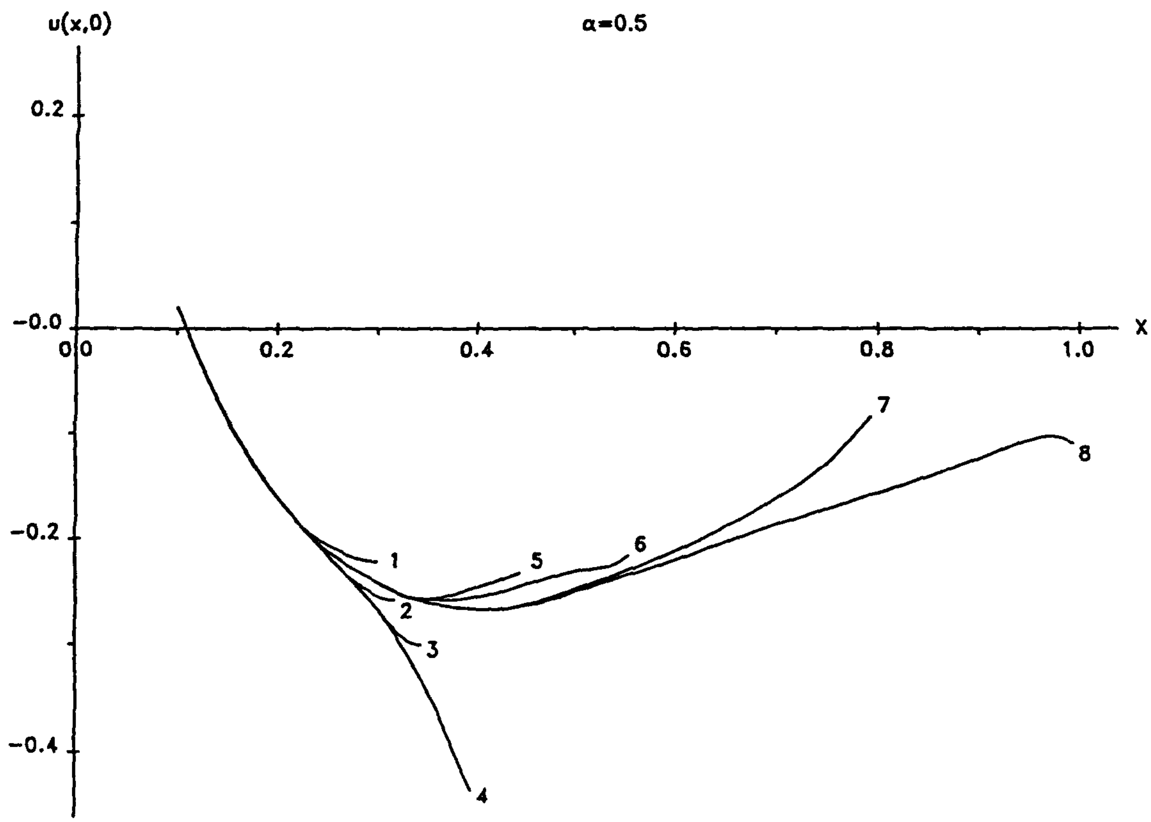

Figure 2

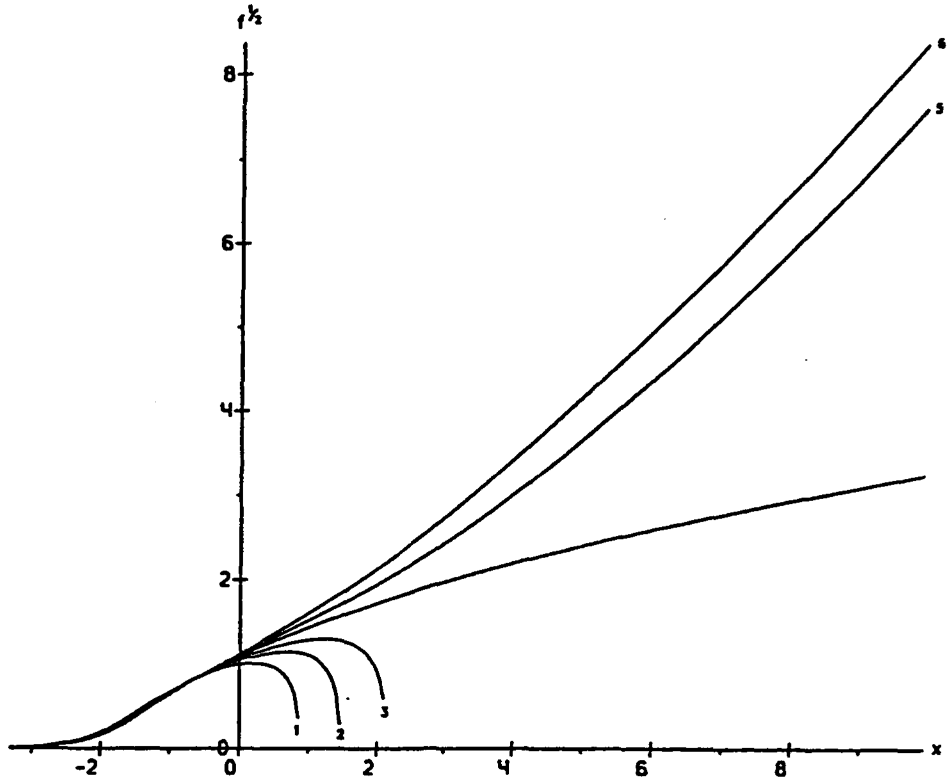

Figure 3

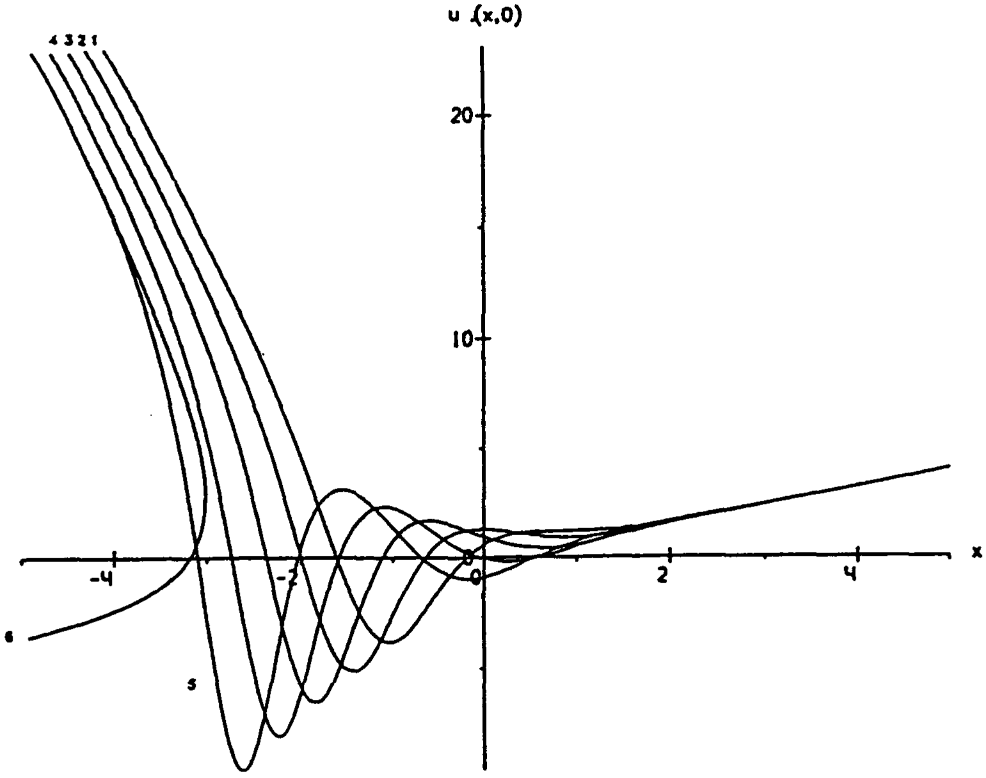

Figure 4

ENERGY TRANSFER FROM A TURBULENT BOUNDARY LAYER MEAN FLOW TO 3D LARGE SCALE WAVES

by

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Whereas stability concepts have commonly been associated to "transition to turbulence" phenomena, it is only since 1956 (Malkus, 1956) that stability and fully developed turbulence have been viewed together. The initial aim was modeling (Malkus, 1956). To this end Malkus assumed marginal stability of his (channel) mean velocity profile, relying on the Orr-Sommerfeld equation. Later, Malkus (1983) stressed the sensitivity of the numerical results to the details of the basic profile.

Other authors, like Reynolds (1972) and Reynolds and Hussain (1972), were interested in large scale waves within turbulent shear layers. Reynolds (1972) discussed the meaning of instability in the context of turbulent mean flows: instability is a criterion for the non-existence of the considered model and, simultaneously, for the existence of the corresponding waves. Reynolds and Hussain (1972), using a triple decomposition, formed an equation for the organized (or wavy) motion energy, in which the perturbed Reynolds stresses are presumed to play a prominent part in a wide region of the wall shear layer.

This bibliography permits understanding of our contribution to this conceptually and technically complex topic. Since the work started by Pace and Dériat (1990), we have been interested in the linear stability study of a turbulent boundary layer mean velocity profile. Our approach uses asymptotic methods; a computational code, as well as a closure model, are necessary in order to go beyond the first approximation.

The first original part of our work is that the asymptotic nature of the basic flow is taken into account. This implies marginal stability, assumed by Malkus (1956), and the main part of the perturbed Reynolds stresses presumed by Reynolds and Hussain (1972) and means that the 'critical layer' is the defect layer.

Then, a mixing-length model is used and a dispersion relation giving the second approximation of the complex phase velocity is solved. We show that it is possible to lower the sensitivity of the results to the details of the basic profile by a numerical artifact; this has to be linked to the difficulty underlined by Malkus (1983). The modes are weakly damped on scales corresponding to inviscid flow.

Nevertheless, we prove that energy is transferred from the mean flow to a part of the considered waves by computing the production term (see below: $R(\alpha, \gamma)$, where α and γ denote the longitudinal and transverse wavelengths, using the boundary layer thickness length scale) of the energy balance equation of the perturbations.

The following step will be the weakly non-linear interaction of 3D waves which are susceptible of resonance.

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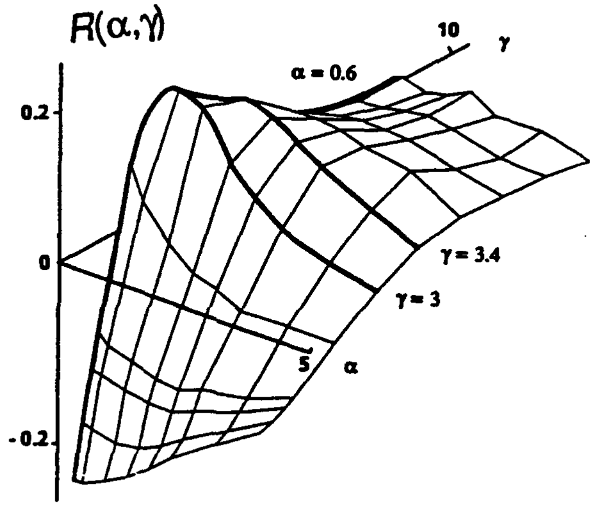

FLOW SEPARATION AND NON-UNIQUENESS OF BOUNDARY-LAYER SOLUTIONS

by

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The solution of the 2-D steady laminar boundary layer equations in presence of an unfavorable pressure gradient is investigated. It is shown that for separation, the solution of the boundary-layer equations behind a point of zero friction is generally not unique. Examples are given as well as procedures to generate such solutions. The non-uniqueness of boundary-layer equations solutions has previously been considered in some self-similar cases (Stewartson, 1954; Smith, 1984).

Let (x, y) be a Cartesian frame of reference, fitted to the body surface, with (u, v) being components of the velocity vector in the (x, y) directions respectively and p is the pressure. The boundary-layer equations and boundary conditions in streamfunction/vorticity formulations are as follows:

$$u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}=\frac{\partial^2\omega}{\partial y^2},\quad u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x},\quad \omega=\frac{\partial^2\psi}{\partial y^2} \tag{1}$$

$$\psi(x,0)=\psi'_y(x,0)=0,\quad \omega(0,y)=\omega_0(y) \tag{2}$$

$$\omega\to 0,\ \text{when}\ y\to\infty \tag{3}$$

$$\frac{\partial\omega}{\partial y}(x,0)=\frac{dp}{dx}. \tag{4}$$

To construct solutions with flow separation and in which the Goldstein singularity is absent, an inverse problem is set first. Instead of the condition (4), one considers a function D(x) defined by

$$D(x)=\int_0^\infty \omega(x,y)y\,dy. \tag{5}$$

The Prandtl equations are solved using this given distribution D(x) and the corresponding p′(x) distribution is found afterwards using a procedure described by Korolev (1987). An example was constructed using the following:

$$\omega_0(y)=(\tau_0+p'_0y^2)\exp[-\alpha y^3(1+\gamma y^2)^{-1}] \tag{6}$$

$$\tau_0=0.7,\quad p'_0=4,\quad \alpha=0.583,\quad \gamma=3.57 \tag{7}$$

$$D'=15,\quad 0\le x\le x_T,\quad D'=15[1+4(x-x_T)^3]^{-1},\quad x_T\le x\le x_c \tag{8}$$

$$D' = -x_2(0.2^2 - x_2^2)^{-1/2}, \quad x_c \le x \le x_E, \quad D' = -1, \quad x \ge x_E \tag{9}$$

$$x_T = 0.33, \quad x_c = 0.51, \quad x_2 = x - x_0, \quad x_0 = 0.71, \quad x_E = 0.85 \tag{10}$$

The solution with a given distribution, $D(x)$ corresponds to a $p'(x)$ distribution, depicted in Figure 1 by a dashed line and a flow pattern with separation and reattachment points (Figure 1, curve 1). Using the same distribution $p'(x)$, we build two more solutions, deviating from the first one by a different distribution of functions behind zero friction points. For this purpose, we again solve numerically the system (1-3), where $p'(x)$ is found from the first solution. However, as a zero approximation, the vorticity of the whole field is taken $w_0(y)$. The numerical solution, coincident with the first one in advance of the separation point changes to a second solution behind the separation point, but the reverse flow region is absent (Figure 1, curve 2).

The third solution differs from the first one downstream of the reattachment point and is constructed in several stages. First, immediately downstream of the reattachment point, one postulates $D(x)$ so as to ensure the slope of the skin friction to be equal in magnitude but opposite in sign to that before the reattachment point; then $p'(x)$ is taken from the first solution. Once convergence is obtained on the first vertical line of the grid downstream of the reattachment point, one further postulates the $D(x)$ close to the value on the previous vertical line; several interactions are carried out, and afterwards $p'(x)$ is used from the first solution. The third solution, obtained in this way, is given in Figure 1 and is denoted as curve 3.

An interesting example of nonuniqueness in the solution of the Prandtl equations is the problem of a flow about the parabolic leading edge of a thin airfoil. The boundary conditions for the boundary-layer equations are as follows. An initial vorticity profile is determined from a condition of a flow stagnation ($w_0(y) = 0$) and the pressure gradient distribution described by (Werle and Davis, 1972)

$$U_e(x) = (Y + k)(Y^2 + 1)^{-1/2}, \quad x = \int_{-k}^{Y} (1 + Y_1^2)^{1/2}\, dY_1, \quad p'(x) = -U_e(x)U_e'(x) \tag{11}$$

The parameter k characterizes the extent of asymmetry of the flow and depends on the angle of attack and the airfoil camber line slope. For $k < k_0$, where $k_0 = 1.17$, the surface friction is positive everywhere and has a minimum. For $k > k_0$ the Goldstein singularity appears in the solution of the equations (Werle and Davis, 1972). At $k = k_0$ the friction minimum approaches zero, and in the vicinity of a zero friction point, a solution is realized which features discontinuously differentiable streamlines (Ruban, 1982) (Figure 2, curve 1). A second solution of the boundary layer equations at $k = k_0$ is constructed analogously to the solution 3 (Figure 1), taking into account the equality of friction slope angles before and after the separation point. The flow described by such a solution (Figure 1, curve 2) unlike the first one, does not contain a reverse flow region, has a separation region, which grows without bound according to $x^{3/4}$ as $x \to \infty$.

Thus the results show that the solution of the boundary-layer equations with continuous boundary-condition evolution and exposed to unfavorable pressure gradients is in general case not unique. The presence of a single zero friction point on the body surface may lead to generation of extra solutions and alternate descriptions of separated flows.

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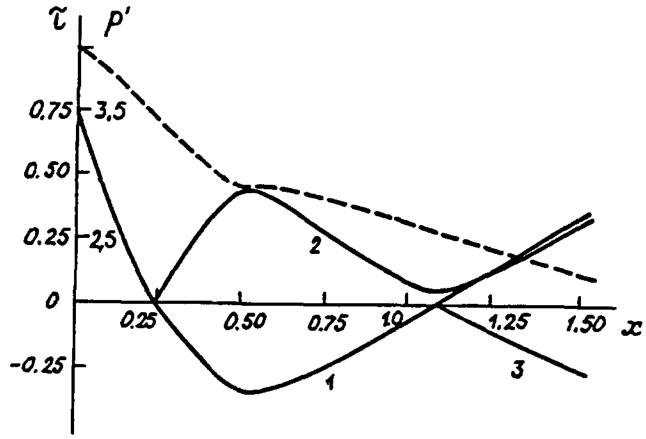

Figure 1

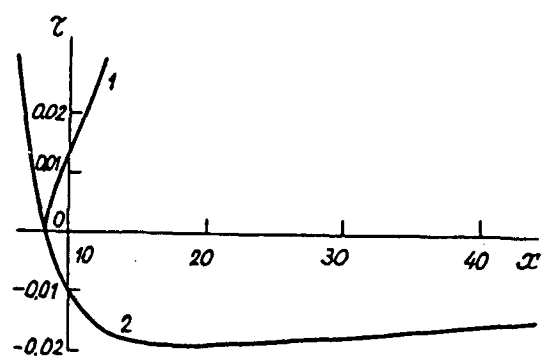

Figure 2

INVISCID-VISCOUS SEPARATION IN UNSTEADY BOUNDARY LAYER SEPARATION

by

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The evolution of the separation process for an unsteady two-dimensional boundary layer at high Reynolds number is considered. It is known that solutions of the classical boundary layer equations often develop a singularity in regions of prescribed adverse pressure gradient. The analytical form of this terminal boundary layer structure for two-dimensional flows was determined by Van Dommelen and Shen (1982) and Elliott, Cowley and Smith (1983). This structure reveals a sharp spike in the boundary layer thickness in a region narrowing in the streamwise direction. As the boundary layer evolves toward the singularity, it bifurcates into two shear layers above and below an intermediate vorticity depleted region which is expanding normal to the surface as the singularity is approached. This terminal state is independent of the specific form of the external adverse pressure gradient responsible for initiating the eruptive process. Consequently, this structure and any subsequent stages are believed to be generic and apply to most cases of unsteady boundary layer separation in two-dimensional incompressible flow.

As the boundary layer starts to separate in a local eruption away from the surface, the external pressure distribution is altered just prior to the formation of the separation singularity due to inviscid-viscous interaction; this is called the first interactive stage and was formulated by Elliott et al. (1983). The three flow regions delineated in the terminal boundary layer structure evolve during this stage subject to an interaction condition relating the external pressure and the growing displacement thickness through a Cauchy principal-value integral. The two shear layers remain passive, while the flow in the intermediate region, governed by the inviscid streamwise momentum equation, is altered by the interaction.

The numerical solution of the first interactive stage was obtained in Lagrangian coordinates; numerical solutions in Lagrangian coordinates have significant advantages over traditional Eulerian formulations for unsteady separation problems of the type considered here. The solution was found to exhibit a high frequency Rayleigh instability resulting in an immediate finite-time breakdown of this stage. When a numerical calculation is attempted, the instability does not permit a grid independent solution to be found, because smaller grid sizes admit shorter wavelength, faster growing modes. The presence of the instability within the formulation of the first interactive stage was confirmed analytically by a linear stability analysis.

GENERATION OF SECONDARY INSTABILITY MODES BY TOLLMIEN-SCHLICHTING WAVE SCATTERING FROM UNEVEN WALLS

by

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It is well-known that the rapidly growing three-dimensional disturbances originate when the amplitude of two-dimensional Tollmien-Schlichting (T-S) waves exceeds a threshold value of approximately 1%. The growth of these disturbances is described by the secondary instability theory developed in Herbert (1985). Nevertheless, the problem of the initial three-dimensional disturbance transformation into the secondary instability modes obtained in Herbert (1985) has not yet been investigated. In this study, the problem is solved for disturbances produced by a plane T-S wave scattering from the unevenness of a streamlined surface. The theory is developed for plane Poiseuille flow, but the results can be easily extended to boundary-layer flow.

Consider the flow of a viscous incompressible fluid in an infinite plane channel. The shapes of the channel walls are given by $Z_w = \pm 1 + \epsilon f_{\pm}(x, y)$ where $f_{\pm}(x, y) \neq 0$ only in the neighborhood of the origin of coordinates. The flow field $V(x, y, z, t)$ may be written as a superposition of the two-dimensional primary flow $V_f(x, z, t)$ and small three-dimensional disturbances $\epsilon V_p(x, y, z, t)$, induced by the unevenness of the walls according to

$$\vec{V} = \vec{V}_f + \epsilon \vec{V}_p$$

where

$$\vec{V}_f = \vec{V}_{fo}(z) + \vec{V}_{fc}(z) \cos(\alpha x - \omega t) + \vec{V}_{fs}(z) \sin(\alpha x - \omega t)$$

$$\vec{V}_p = \vec{V}_o(x, y, z) + \vec{V}_c(x, y, z) \cos(-\omega t) + \vec{V}_s(x, y, z) \sin(-\omega t)$$

and

$$\vec{V}_o = \left\{u_o, v_o, w_o\right\} \quad \vec{V}_c = \left\{u_c, v_c, w_c\right\} \quad \vec{V}_s = \left\{u_s, v_s, w_s\right\}.$$

The primary flow is a combination of Poiseuille flow and a finite-amplitude T-S wave which is taken to be strictly periodic in space and time. After substitution in Navier-Stokes equations, linearization in ϵ, and some algebraic transformations, a linear set of equations with periodic coefficient for w_o, w_c, w_s, η_o, η_c, η_s is obtained. The functions η_o, η_c, η_s are the shape functions for the vertical vorticity component η, viz.

$$\eta = \eta_o(x, y, z) + \eta_c(x, y, z) \cos(-\omega t) + \eta_s(x, y, z) \sin(-\omega t).$$

The boundary conditions are applied at the walls $z = \pm 1$ through linearization of the no-slip condition at the walls. We will assume that all disturbances tend to zero as $y \to \pm\infty$ and $x \to -\infty$. No boundary conditions for $x \to +\infty$ are set.

In the subsequent discussions, we assume that the unevenness is symmetric about the plate at $y=0$, i.e. $f_\pm(x, y) = f_\pm(x, -y)$. If such an assumption is made, one can readily see that w_o, w_c, w_s are even and η_o, η_c, η_s are odd functions of y.

Fourier transforms of the equations and boundary conditions are taken, with cosine- and sine-Fourier transforms in spanwise direction being used for the even and odd functions. A complex Fourier transform in the streamwise direction is used for all functions. The Fourier transforms of the functions u, v, w, η, $f_\pm$ will be denoted by U, V, W, H, $F_\pm$ and k, and β will be used for the streamwise and spanwise Fourier variables. After transformation, a set of linear ordinary differential equations for functions W_o, W_c, W_s, H_o, H_c, H_s of k, $k+\alpha$ and $k-\alpha$ is obtained. The functions of the last two arguments appear as a result of the Fourier transforms of the terms, including the periodic coefficients.

The solution of this set of equations is difficult due to presence of the same functions but with different arguments. These difficulties may be avoided if we assume that longitudinal size of the unevenness ℓ is large in comparison with the wavelength of the primary wave. In this case the problem is reduced to the set of equations for functions W_o, H_o, W, C_{c1}, W_{s1}, H_{c1}, H_{s1} of unique argument k, which may be solved numerically. (Here $W_{c1}(k) = W_c(k-\alpha)$, W_{s1}, H_{c1}, H_{s1} are defined analogously).

Let us assume solutions of the above problem to be analytical functions of complex variable k. If all poles of these solutions lie in the upper half of the complex k-plane, the classical expression for the velocity disturbances given by inverse Fourier transformation is valid. If one or more poles are located below the real axis, such a solution is not consistent with the physical meaning of the problem discussed. As shown in Bogdanova and Ryzhov (1982), the special terms defined by the residues of these poles should be added to the classical solution to obtain the correct solution. Because the classical solution tends to zero as $x \to +\infty$, disturbances at a large distance downstream of the unevenness are determined by the exponentially growing additional terms associated with the secondary instability modes. When the primary wave amplitude is sufficiently large, the two poles in the lower part of complex k-plane located on the imaginary axes are found numerically. The pole, associated with the most unstable secondary instability mode, corresponds to even function $W_o(z)$; the second pole is associated with the mode having an odd function $W_o(z)$. By choosing asymmetrical boundary conditions $f_-(x, y) = f_+(x, y)$, the second pole is removed and only the pole located at the point $k_o = -i\sigma(\beta)$ remains. The amplification rate σ as a function of β for disturbances associated with this pole is shown in Figure 1. Curves 1 and 2 correspond to primary T-S wave amplitudes $a = \max\limits_z \sqrt{u_{fc}^2 + u_{fs}^2} = 0.0182$ and 0.0343.

In accordance with the above discussion, the following expression for the velocity disturbances at large distances from the unevenness may be written

$$u(x, y, z, t) = u_o(x, y, z) + u_a(x, y, z)e^{i(\alpha x - \omega t)} + u_a^*(x, y, z)e^{-i(\alpha x - \omega t)}, \tag{1}$$

where

$$u_o = \sqrt{\frac{2}{\pi}} \int_0^\infty u_{so}(\beta, z)e^{\sigma(\beta)x} \cos \beta y \, d\beta, \quad u_a = \sqrt{\frac{2}{\pi}} \int_0^\infty u_{sa}(\beta, z)e^{\sigma(\beta)x} \cos \beta y \, d\beta, \tag{2}$$

and

$$u_{so} = -2\pi i \frac{\beta}{k_o^2} \operatorname*{res}_{k=k_o} H_o, \quad u_{sa} = \frac{2\pi i}{k_o^2+\beta^2}\left[i(k_o+\alpha) \operatorname*{res}_{k=k_o} W'_{c1} - \beta \operatorname*{res}_{k=k_o} H_{c1}\right]. \tag{3}$$

If the longitudinal and spanwise extent of the unevenness are small enough, $F_\pm(k_o, \beta)$ are proportional to the unevenness volume. Since residues of all functions in equation (1) are proportional to $F_{\pm}(k_o, \beta)$, the velocity disturbances from such an unevenness should be proportional to its volume.

The presence of maximum of $\sigma(\beta)$ in $\beta = \beta_o$ allows us to use the Laplace method for the evaluation of integrals in equations (2) as $x \to +\infty$ and

$$u_o \approx A(x, y)\, u_{so}(\beta_o, z) \cos \beta_o y \qquad u_a \approx A(x, y)\, u_{su}(\beta_o, z) \cos \beta_o y \tag{4}$$

$$A(x, y) = \sqrt{\frac{2}{a_1 x}} \exp\left(a_o x - \frac{y^2}{4a_1 x}\right) \qquad a_o = \sigma(\beta_o) \quad a_1 = -\frac{1}{2}\frac{d^2\sigma}{d\beta^2}(\beta_o). \tag{5}$$

Here $A(x, y)$ determines the disturbance amplitude and a curve $A(x, y) = C = \text{constant}$ may be treated as the boundary of the disturbed region. The equation of this line is

$$y = 2\sqrt{a_o a_1}\,|x|\sqrt{1 - \frac{c}{a_o x}}\,.$$

As $x \to +\infty$ this boundary line approaches a wedge with apex angle given by

$$\alpha = 2 \text{ arctg } (2\sqrt{a_o a_1}). \tag{6}$$

This angle as a function of the primary wave amplitude is shown in Figure 2. The shape of disturbed domain obtained here is similar to the turbulent wedge appearing downstream of a surface hump in experiments.

In deriving asymptotic formulae (4) – (6), only the existence of the pole in the lower part of complex k-plane and the form of $\sigma(\beta)$ dependence are used. The position of this pole is independent of the unevenness shape and dimensions and is defined by the amplification rate of the secondary instability mode. Consequently, (4) – (6) are valid for short unevennesses with $\ell \ll \alpha^{-1}$ also. For such an unevenness, the proportionality of velocity disturbances to unevenness volume is valid, but the proportionality coefficient cannot be determined by the method presented above.

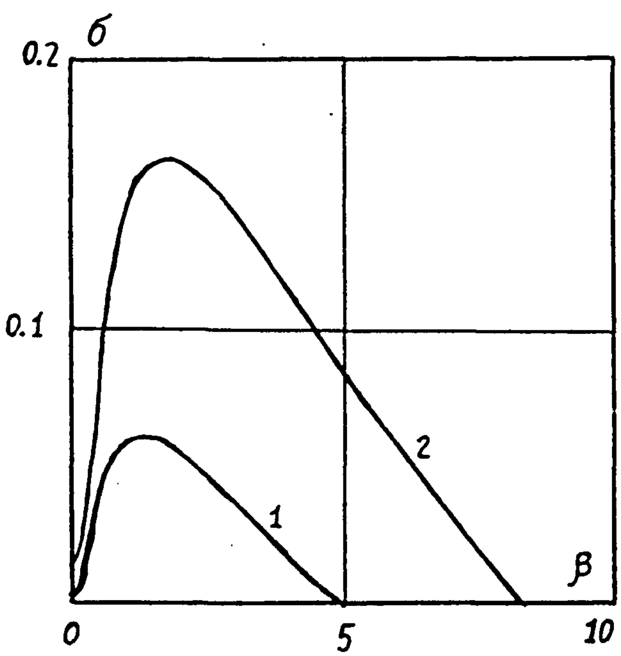

Figure 1

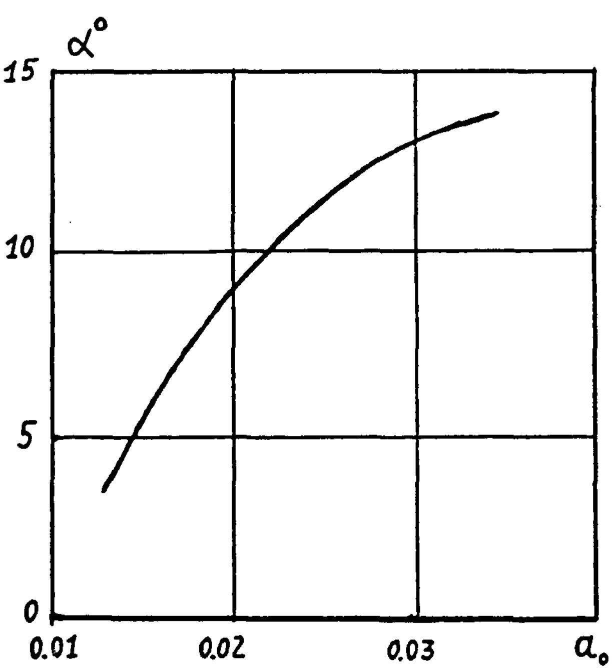

Figure 2

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OBLIQUE INSTABILITY WAVES IN NEARLY PARALLEL SHEAR FLOWS

by

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Asymptotic methods are used to describe the nonlinear self-interaction between a pair of oblique instability waves that eventually develop when initially linear, spatially growing instabilities evolve downstream in nominally two-dimensional unbounded or semi-bounded, laminar shear flows. The Reynolds number is assumed to be large enough so that the flow is nearly parallel, and we suppose that some sort of small-amplitude harmonic excitation (i.e., an excitation of a single frequency) is imposed on the flow. The initial motion just downstream of the excitation device then has harmonic time-dependence and, within a few wavelengths or so, is well described by linear instability wave theory.

In highly unstable flows, such as free shear layers, jets, and (separated or unseparated) wall boundary layers with O(1) adverse-pressure gradients, the peak linear growth rate is of the same order as the inverse shear-layer thickness Δ^{-1}, but in the more stable flows, such as flat-plate boundary layers or unseparated boundary layers with relatively weak adverse-pressure gradients, the peak growth rate is typically small compared to Δ^{-1}. The latter case usually obtains for most wall boundary layers which remain unseparated.

However, even in flows where the peak linear growth rate is $O(\Delta^{-1})$, the local growth rate is usually small (relative to Δ^{-1}) by the time nonlinear effects set in because mean-flow divergence effects tend to cause the growth rate to decrease as the instability wave propagates downstream.

We begin by considering the initially linear stage just downstream of the excitation device. In some flows, such as a supersonic free shear layer, or a flat-plate boundary layer in the relatively low, supersonic Mach-number regime, where the so-called first-mode instability is dominant, the most rapidly growing mode is an oblique wave, so that the oblique mode self-interaction that is of interest herein is likely to be the first nonlinear interaction to occur. However, in most flows, it is the plane wave that exhibits the most rapid growth, and some intermediate interaction must occur before the oblique modes can interact with themselves. This intermediate stage can involve the parametric interaction of the oblique modes with the plane wave or with some pre-existing spanwise distortion of the mean flow. This stage can be treated simultaneously with the self-interaction stage if we begin by considering a resonant triad of instability waves imposed on a slightly distorted mean flow in the initial linear region – a plane fundamental frequency wave and a pair of oblique equi-amplitude subharmonic waves, with the same streamwise wave number and frequency but with equal and opposite spanwise wave numbers. The latter two waves combine to form a standing wave in the spanwise direction that propagates only in the direction of flow – which is the situation that most frequently occurs in wave excitation experiments that typically involve relatively long excitation devices placed perpendicular to the flow.

Since our scaling requires that the instability-wave growth rate be small in the nonlinear region of the flow, and since the Reynolds number is assumed to be large, the first nonlinear reaction to occur must take place locally within the so-called "critical layer" where the mean-flow velocity is equal to the common phase velocity of the two or three modes that interact there. The flow outside the critical layer is still governed by linear dynamics, which means that it is given by a slightly distorted locally parallel, mean flow, plus a pair of oblique instability-wave modes plus a plane wave.

The instability wave amplitude (or amplitudes) is (or are) completely determined by nonlinear effects within the critical layer. However these nonlinearities are weak in the sense that they enter through inhomogeneous terms in a higher order problem, rather than through the coefficients in the lowest order equations. The instability wave amplitude (or amplitudes) is (or are) then determined by an integral differential equation (or a pair of integral differential equations) with quadratic to quadic-type nonlinearities. The most important feature of this equation is the oblique mode self-interaction terms that eventually lead to a singularity at a finite downstream position. It is shown that this type of interaction is quite ubiquitous and is the dominant nonlinear interaction in many apparently unrelated shear flows, even when the oblique modes do not exhibit the most rapid growth in the initial linear stage.

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